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INTERACTION OF THE SOLAR PLASMA WITH THE EARTH'S MAGNETIC FIELD*

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Recent satellite observations of the interplanetary magnetic field[1,2] and the belts of trapped radiation surrounding our planet have focused attention on the shape and extent of the terrestrial magnetic field. The terrestrial magnetic field, if it occurred in a vacuum, could be well approximated by a simple magnetic dipole field. Many authors, including Bierman[3] in particular, however, have pointed out that the sun perpetually emits an intense time-varying neutral flux of energetic protons and electrons. It is the purpose of this Letter to show how the magnetic field surrounding the earth is reshaped and terminated by the incident current of what amounts to a diamagnetic medium.

The charged particles in a flow of neutral plasma will follow different paths on encountering a magnetic field depending on their charge and mass. The electrons having negligible momentum are deflected by the magnetic field in an infinitesimal distance when compared with the proton behavior creating a large electric field due to charge separation. The electric field decelerates the protons and accelerates the electrons and their combined motion results in a very thin current layer beyond which the total magnetic field is zero. By adapting Rosenbluth's[4] treatment of the magnetic pinch to the problem, the thickness of the layer was found to be of the order of $(8\pi N_0 e^2/m_e c^2)^{-1/2}$, where N_0 is the particle density in the plasma, e the particle charge, c the velocity of light, and m_e the mass of the electron. For $N_0 = 50/\text{cc}$ the sheath thickness amounts to 0.3 kilometer.

Under such circumstances it has been shown that the hydromagnetic approximation,

$$p + B^2/8\pi = \text{constant} \tag{1}$$

(where p is the proton pressure, a tensor, and B is the magnetic field), is valid except for a small region very close to the magnetic poles. For proton mass m_i and initial plasma velocity $\vec{v}$,

$$p = 2N_0 m_i v^2 \cos^2\psi, \tag{2}$$

where ψ is the angle between $\vec{v}$ and the normal to the surface of the current sheath. In a spherical coordinate system where the $\theta = 0$ axis is parallel to the earth's dipole and $\vec{v}$ is in the plane of $\phi = \pi/2$,

$$\cos\psi = (\vec{v}\cdot \text{grad}F)/(v\,|\,\text{grad}F\,|) = a\left\{\frac{\partial F}{\partial r}[\sin\phi\,\sin\theta\,\cos\lambda - \cos\theta\,\sin\lambda]\right.$$

$$\left. + \frac{1}{r}\frac{\partial F}{\partial \theta}[\sin\phi\,\cos\theta\,\cos\lambda + \sin\theta\,\sin\lambda] + \frac{1}{r\sin\theta}\frac{\partial F}{\partial \phi}[\cos\phi\,\cos\lambda]\right\}, \tag{3}$$

Table I. Values of α and β as functions of θ, the latitude angle, where α and β are the coefficients in a series expansion of r of the plasma sheath surface as a function of η, the longitudinal angle from the plane containing the earth's dipole and the earth-sun line. $r = 1 + \alpha\eta^2 + \beta\eta^4$ in units of $r_0 = (M^2 8\pi N_0 m_i v^2)^{1/6}$, and η is expressed in radians.

θ	90°	75°	60°	45°	30°	15°
α	0.104	0.102	0.096	0.086	0.068	0.031
β	0.011	0.011	0.010	0.008	0.007	0.004

where a is a normalizing factor involving partial derivatives, λ is the solar latitude position, and $F(r, \theta, \phi) = \text{constant}$ is the equation for the surface of the current sheath.

In the case of a plane current sheath, only the local surface current would contribute to the component of the magnetic field tangential to the sheet. In the case of the curved surface derived in this work, neglecting the nonlocal currents is only an excellent first approximation. The surface derived here should be used to derive a closer approximation to the exact magnetic field in higher approximation. The magnetic field resulting from a hemispherical current sheath whose current density is an appropriate function of angular position has been calculated and found to be twenty to forty percent of the total magnetic field at the sheath, depending on the position. Since the earth's magnetic field decreases as the cube of the distance, the surface is brought in 6% closer to the earth. The shape of the surface is altered slightly; the radial distance of the sheath everywhere being less, relative to the sheath intersection with the earth-sun line, than calculated in first approximation.

The component of the earth's magnetic field parallel to the sheath surface is doubled on the interior by the surface current and is canceled on the exterior of the surface. The component of the earth's field perpendicular to the surface is canceled on both sides of the sheath surface in the neighborhood of the sheath by the changes in surface current that occur along a magnetic field line. The component of the earth's field which lies in the surface sheath is given by

$$B_S = (Mb/r^3)[\sin\theta + 2\cos\theta(dr/d\theta)], \qquad (4)$$

where b is another normalizing factor, M is the earth's dipole moment, and the derivative is taken in the plane of the sheath surface.

The differential equation for the surface of the sheath resulting from substituting Eqs. (2), (3), and (4) into Eq. (1) has been solved approximately. For the daylit side of the earth and $\lambda = 0$ the solution is well approximated between $\phi = \pi/2$ and $\phi = \pi$ by a power series in η, where $\eta = \phi - \pi/2$:

$$r = 1 + \alpha\eta^2 + \beta\eta^4; \qquad (5)$$

α and β are functions of θ. r is expressed in units of $r_0 = (M^2 8\pi N_0 m_i v^2)^{1/6}$. r_0 represents a scaling factor dependent on the solar plasma pressure. If $N_0 = 50/\text{cc}$ and $v = 2\times10^8$ cm/sec, r_0 would be about five earth radii which is approximately the position of the sudden change in the earth's field reported by Sonnett et al.[1] from satellite observations. Values of α and β as a function of θ are presented in Table I. Table II presents values of r and $|r\cos\phi|$ as a function of ϕ for $\theta = 90°$. Table III presents approximate values of r as a function of θ for $\phi = 3\pi/2$, the night side of the earth where the sheath current is reversed (at $\phi = 3\pi/2$) over the daylit current direction. A more detailed discussion and complete calculation will be presented elsewhere.

The predictions of infinite r on the night side of the earth should not be taken seriously since sidewise pressure from a radial solar magnetic field, or especially sidewise proton pressure from a nonzero temperature plasma, would close the "tail" of the sheath as expected from Eq. (1). The axis of the sheath surface is shifted roughly one third of λ for $\lambda \neq 0$; otherwise the surface is affected little by changes in λ.

Table II. r of the plasma sheath surface (in units of r_0) and $|r\cos\phi|$ as functions of ϕ, the longitudinal position in the equatorial plane of the earth's magnetic dipole. $180° < \phi < 270°$ corresponds to the night side of the earth.

ϕ	90	105	120	135	150	180	210	240	270
r	1.000	1.009	1.031	1.068	1.126	1.342	1.842	3.472	∞
$\lvert r\cos\phi\rvert$	0	0.261	0.515	0.755	0.976	1.342	1.595	1.736	1.781

Table III. r of the plasma sheath surface (in units of r_0) on the night side of the earth and $|r \sin\theta|$ as functions of polar angle in the plane of the earth's dipole and earth-sun line.

θ	15.5°	19°	23°	37.5°	55°	69°	90°
r	~1.10	1.26	1.50	2.00	3.00	5.00	∞
$\lvert r \sin\theta \rvert$	~1.06	1.19	1.38	1.58	1.72	1.77	1.785

It is a pleasure for the author to have this opportunity to thank Dr. Alex Dessler and especially Dr. Francis S. Johnson, both of Lockheed, for bringing the problem to the author's attention, and for much illuminating discussion.

*This work was undertaken while the author was a consultant for the Lockheed Missiles and Space Division.

[1]C. P. Sonnett, E. J. Smith, D. L. Judge, and P. J. Coleman, Jr., Phys. Rev. Letters 4, 161 (1960).
[2]M. Neugebauer, Phys. Rev. Letters 4, 6 (1960).
[3]L. Bierman, Observatory 77, 109 (1957).
[4]M. Rosenbluth, in Magnetohydrodynamics, edited by R. Landshoff (Stanford University Press, Palo Alto, California, 1957), p. 57.

EVIDENCE FOR A CONFIGURATIONAL EMF IN A CONDUCTING MEDIUM*

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There is a pressure drop associated with an increase in the flow rate of a moving fluid. The venturi meter and the aspirator are familiar devices which derive from this phenomenon. This paper concerns itself with the detection of an analogous effect in the electron gas in a metal.

The usual analysis for the pressure drop in classical fluids requires the conservation of energy. Associated with a higher flow rate is an increase in kinetic energy. The pressure is a measure of potential energy. It must be lower in the region of higher flow velocity where the kinetic energy is higher. If the higher flow velocity is brought about by a narrowing of the flow channel then the phenomenon is called the Bernoulli effect.

In the electron gas of a metal the dissipative effects dominate over the inertial ones. Unlike the case for a viscous fluid, the dissipation comes from the transfer of momentum to the lattice vibrations and imperfections rather than to the walls of the container. Regardless of the nature of the collisions, however, the two fluids have much in common. They both have need of the presence of a force merely to maintain the flow. A force maintains the current in the face of the scattering of forward momentum into random motion by collisions. In the electron fluid this force is the applied electric field. By Ohm's law the force is proportional to the flow velocity. The electric field in the electron gas is the analog of the pressure gradient in viscous fluid flow.

The electron gas in a metal sees a background of positive charge from the ion cores. These keep the electron density uniform on penalty of building up high internal fields in the metal. The electron gas is, therefore, relatively incompressible.

Suppose that an electric current flow is constricted in cross section as in Fig. 1. Continuity and incompressibility demand that the flow velocity or drift of electrons be greater in the constricted region. Inspired by analogy with the hydrodynamic problem, the following question is posed: What accelerates the electrons in the region of changing flow rate so that they acquire the increased speed necessary for drift through the constriction? Is there an additional field present from which the electrons get the extra accelerating kick which they must have in order to move faster in the constriction? The experi-

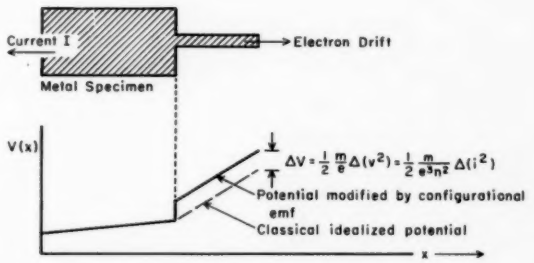

FIG. 1. Expected potential vs distance due to constricted current flow in the conducting specimen shown.

mental objective was the detection of the potential difference associated with such an accelerating field.

The existence of such a potential reflects the inexactness of Ohm's law. The effect of this non-Ohmic potential is shown diagramatically in Fig. 1. The potential step corresponding to the spatial step is the same regardless of the direction of current flow. This is in contrast to a constriction resistive potential which reverses with current.

There is a parallel structure in the analysis of the Bernoulli effect and of the configurational emf. It seems reasonable, therefore, to make a first estimate of the configurational voltage from the following equation:

$$-e\Delta V_c + \tfrac{1}{2}m\Delta v^2 = 0 \qquad (1)$$

Here V_c is the configurational potential, e is the electronic charge, m is the electronic mass, and v is the drift velocity. When the constricted area is sufficiently smaller than the area of the wider flow region, Eq. (1) may be written:

$$\Delta V_c = \tfrac{1}{2}(m/e^3n^2)i^2. \qquad (2)$$

Here i is the current density in the constricted region and n is the number of current carriers per unit volume.

The essential quality which makes the configurational emf detectible is its dependence on the square of the current. It has the same polarity regardless of the direction of current flow. This polarity depends only upon the sign of the charge of the current carriers. Equations (1) and (2) apply to negative carriers. In principle, the experiment requires only that an ac signal be applied to a stepped specimen like that of Fig. 1. A dc voltmeter across the step then reads the configurational emf directly.

The experiment was carried out in thin-film samples of bismuth. These were shaped basically as shown in Fig. 2. Note that all foreign wire leads from the external circuitry meet the sample material only at regions thermally removed from the central strip carrying the high current density. This procedure avoids introducing thermocouple emf's into the circuit. A special technique employing a bridge circuit with the specimen of Fig. 2 insured that the signal was not due to rectifying contacts at the specimen terminals.

A definite unidirectional configurational voltage was detectible. This voltage was proportional to the square of the current out to current densities estimated as high as 10^6 amperes/cm^2. Some of the specimens appeared to show conduction by positive carriers. This was deduced from the sign of the configurational voltage. In each case the carrier sign was confirmed by a Hall effect measurement.

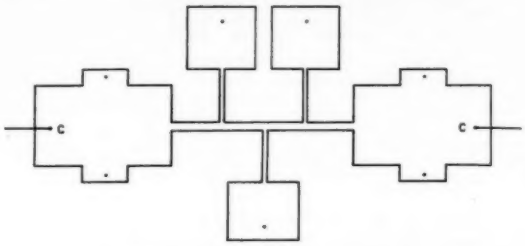

FIG. 2. Evaporated thin film specimen geometry. Points C are current terminals; all other points are potential contacts.

Various supplementary experiments shows that the detected voltage was clearly related to the configuration of the specimen. For example, a null effect results if the two potential contact points do not span a change in cross section. Figure 2 shows the extra potential probes provided for consistency tests of this kind.

The positive carrier specimens gave large signals but noisy and erratic ones. They were poor specimens on which to make measurements. The negative carrier specimens gave consistent and reproducible results.

The quantitative correlation between the experimental data and Eq. (2) was quite poor. The calculations were made using the value of the carrier density, n, obtained from direct Hall measurements on the samples.

That there is quantitative disagreement with Eq. (2) is not surprising. The crudeness of its derivation suggests that at best it might be applicable to an ideal metal like sodium or potassium.

Recently more elaborate and detailed calculations have been made. The analysis makes use of the Boltzmann transport equation as applied to the Fermi gas of electrons. The results indicate that in certain cases a better statement than that of Eq. (2) is given by

$$\Delta V_c = \gamma(m/m^*)(m/e^3n^2)i^2. \qquad (3)$$

The factor γ is of order unity. It involves integrals over the angular distribution of the phonon

scattering cross section. The quantity m^* represents an average effective mass of electrons at the top of the Fermi sea.

Equation (3) suggests that the discrepancy between the experimental data and Eq. (2) arises from the deviation of m^*/m from unity in bismuth. If so interpreted, the data indicate that the value of this ratio is of the order $m^*/m = 10^{-4}$.

In summary then, there is little doubt empirically that there is an emf associated with a current flow constriction. Those gross properties of this emf which have been investigated seem to correlate well with those derived from the hydrodynamic principles which motivated this research. Although this explanation of the configurational emf appears to be the most reasonable one at present, it has not been totally verified. Further investigation on a variety of materials has been initiated to resolve the matter. It is hoped that new experimental results will be available for presentation soon to supplement the extended theoretical analysis which is in preparation.

The author is indebted to Professor John R. Pellam for many helpful discussions and to Professor Richard P. Feynman for theoretical insights.

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SHALLOW IMPURITY TRAPS AND ELECTRON TRANSFER DYNAMICS IN n-TYPE SILICON AT LIQUID HELIUM TEMPERATURES*

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From combined electron spin resonance and photoconductivity experiments on samples of phosphorus-doped silicon with boron compensation, we have been able to determine some interesting properties of shallow impurities in silicon. These include (a) rate of electron transfer[1] from a neutral phosphorus impurity to a neutral boron impurity at various impurity concentrations and temperatures, and (b) ratio of positive phosphorus ion trapping cross section to neutral boron trapping cross section for conduction electrons. In addition, the technique we introduce allows an accurate determination of the ratio of donor concentration to compensating acceptor concentration without recourse to transport phenomena. The technique also permits controlling the number of ionized impurities in a given sample while at low temperatures (liquid helium); this makes it possible to investigate in a simple manner physical properties which may depend on ionized impurity concentration, such as low-temperature mobility. Information on the lifetimes of hot carriers as a function of their energy can also be obtained.

We now describe the experimental procedure and theory relevant to the above studies. Consider a sample of n-type silicon containing P phosphorus impurities/cm^3 and B boron impurities/cm^3. In equilibrium at liquid helium temperatures, the neutral phosphorus concentration P^0 equals $(P - B)$, the positive phosphorus ion concentration P^+ is equal to B, and the negative boron ion concentration B^- also equals B. The P^0 contain unpaired electrons, and thus the electron spin resonance signal[2] is proportional to P^0. We now produce electron-hole pairs uniformly throughout the sample by illuminating with intrinsic radiation (1.03 microns)[3] near the indirect transition threshold. The electrons and holes are rapidly captured; the electrons predominantly by the P^+ ions, and the holes predominantly by the B^- ions. If the rate of generation of electron-hole pairs is much greater than the rate of electron transfer from the P^0 to the B^0, then the P^0 concentration builds up very close to P. The new resonance signal is thus $(1 - B/P)^{-1}$ times the original resonance signal, and the compensation B/P is directly determined. If the intrinsic radiation is now turned off and the resonance signal monitored, the rate of electron transfer from P^0 to B^0 can be determined. The electron transfer rate can be increased greatly by delocalizing the electrons with extrinsic radiation. A wavelength of about 2 microns insures a strong predominance of free electrons over holes because of the free-carrier absorption peak[4] in n-type silicon. If we consider a simple model in which P^+ and B^0 are the

only electron traps, then the following kinetic equations characterize the decrease of P^0 as it approaches its equilibrium value of $(P - B)$:

$$dP^0/dt = -SP^0 + pP^+n_e,$$

$$dn_e/dt = SP^0 - pP^+n_e - bB^0n_e. \qquad (1)$$

S is the rate of extrinsic ionization of the P^0, p and b are, respectively, the capture probabilities for a conduction electron of P^+ and B^0, and n_e is the conduction electron concentration. The solution of Eq. (1) gives P^+ or P^0 $(= P - P^+)$ as a function of t, as well as n_e as a function of t. These solutions are

$$\frac{2bSt}{p} = \ln\left\{\left(\frac{PB - (P+B)P^+ + (P^+)^2}{PB - (P+B)P_i^+ + (P_i^+)^2}\right) \times \left[\frac{(P^+ - P)(P_i^+ - B)}{(P_i^+ - P)(P^+ - B)}\right]^{(P+B)/(P-B)}\right\}, \qquad (2a)$$

$$n_e = S(P - P^+)/(pP^+ + bB^0). \qquad (2b)$$

P_i^+ is the value of P^+ at $t = 0$. In Fig. 1(a), P^0 vs $2(b/p)St$ is plotted from Eq. (2a). Since S can be directly determined from the electron interchange spin relaxation time[5] associated with the two phosphorus hyperfine lines, we are able to determine b/p by fitting the theoretical curve to the experimental values of P^0 obtained by spin resonance. In Fig. 1(b), n_e vs $2(b/p)St$ is plotted from Eq. (2b), and the experimental photoconductive current also appears. If the mobility remained constant and the simple model we have chosen were correct, the two curves would coincide. The discrepancies will be discussed below.

The same model used above also predicts the time dependence of P^0 and n_e during the P^+ trap-filling process with the intrinsic radiation on. In Fig. 2(a), the experimental and theoretical curves of P^0 vs time are seen. P^0 increases essentially linearly with time at a rate corresponding to the rate of electron-hole pair production. In Fig. 2(b), n_e and the observed photocurrent are plotted against time during the trap filling process. Agreement between these last curves depends on the mobility constancy, the adequacy of the model, and the inclusion of hole current. Electron current dominates even under intrinsic excitation, as seen from photo-Hall measurements, but it is probable that the hole current must also be taken into consideration in this case.

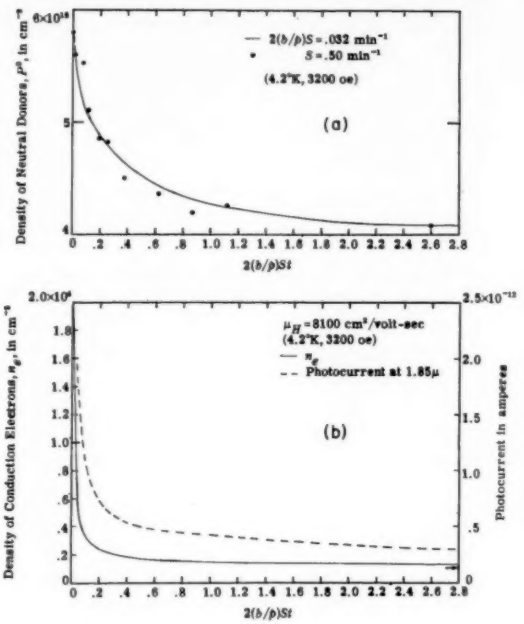

FIG. 1. (a) Decay of neutral phosphorus concentration P^0 under 1.85-μ extrinsic radiation, after the P^+ traps are filled with electrons. Experimental points are obtained from spin resonance measurements, and solid curve gives theoretical decay based on Eq. (1), with value of b/p chosen for best fit. (b) Decay under same conditions as (a) of conduction electron concentration n_e based on Eq. (1) and of observed photocurrent. Arrow denotes extrinsic photocurrent for equilibrium trap occupancy. μ_H remains fairly constant from photo-Hall measurements. The silicon sample contains 6.0×10^{15} P/cm^3 and 2.0×10^{15} B/cm^3.

The experiments were performed on a sample of silicon containing nominally 5×10^{15} P/cm^3 and 2.5×10^{15} B/cm^3. A combination of Hall effect measurements and experiments of the type discussed above which yield B/P [see Fig. 2(a)] indicate actual concentrations of 6.0×10^{15} P/cm^3 and 2.0×10^{15} B/cm^3. The electron transfer time from P^0 to B^0 in the absence of extrinsic radiation from the monochromator exceeds 15 minutes at both 4.2°K and 1.3°K. It may be considerably longer than 15 minutes, since a small amount of leakage radiation is probably present. The b/p ratio which is obtained from Fig. 1(a) is 0.032, to an accuracy of about 50%. It is temperature independent between 1.3°K and 4.2°K. The discrepancies between the n_e and photocurrent curves in Figs. 1(b) and 2(b) are believed to be

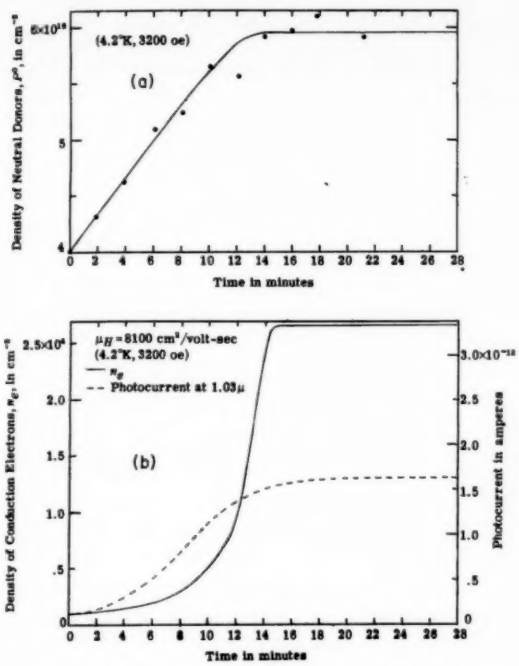

FIG. 2. (a) Growth of neutral phosphorus concentration under 1.03-μ intrinsic radiation, due to the process $P^+ + e \rightarrow P^0$. Experimental points are obtained from spin resonance measurements. Solid curve gives theoretical growth. (b) Growth of intrinsic photocurrent under same conditions as in (a). Theoretical growth of n_e is given by solid curve. The silicon sample contains 6.0×10^{15} P/cm^3 and 2.0×10^{15} B/cm^3.

due to the inadequacy of the model used. The low-temperature mobility as measured by photo-Hall effect appears to be fairly constant as the P^+ traps are filled, and yet a large photocurrent increase results from a relatively small decrease in impurity ion concentration. The data suggest that at high ion concentrations, the lifetime against trapping for 2-micron extrinsically excited electrons depends more nearly on $(1/P^+)^2$ rather than on $(1/P^+)$, which was assumed in our model. The total enhancement of the photocurrent, i.e., the ratio of the photocurrent when the maximum number of traps are filled to the photocurrent for the equilibrium trap electron occupancy, is smaller than predicted by our model, indicating the presence of other types of lifetime-limiting traps.[6] These may be deep traps, or possibly P^0 traps which can form stable P^- centers, the analog of the bound hydrogen negative ion. A photoconductivity experiment with highly spin-polarized electrons, which would not be expected to be trapped by similarly polarized P^0 centers, was inconclusive.

The extrinsically (2-micron) generated electron's lifetime against trapping, for the equilibrium trap situation where $P^+ = B$ (2×10^{15}/cm^3 for the sample under discussion), is determined from the expression $\tau = n_e/SP^0$, where n_e is measured by Hall effect and S is the rate of ionization of P^0, which is determined from the interchange spin relaxation time[5] as mentioned above. At 4.2°K and 1.3°K, we obtain $\tau \cong 5\times10^{-11}$ sec. If about 20-micron extrinsic radiation is used, τ is somewhat decreased by less than a factor of 2, and μ_H is decreased by a factor of about 4, compared with the 2-micron generated electrons. This suggests that an appreciable part of the lifetime of the 2-micron extrinsically generated electrons is spent in thermalization. The 20-micron excited electrons are not as "hot," and apparently thermalize and get trapped more quickly.

In an experiment performed with a sample of silicon nominally containing 5×10^{16} P/cm^3 and 2.5×10^{16} B/cm^3, kindly lent us by Dr. Ludwig of General Electric Research Laboratories, the dark conductivity at 4.2°K decreased by a factor of 5 upon irradiating with intrinsic light. We attribute this to the filling of the P^+ traps, and the attendant decrease in impurity band conduction which results from lowering the concentration of ions.

We would like to thank Dr. Bruce Rosenblum of R. C. A. Laboratories for having kindly provided the principal silicon sample used in these experiments and for an early discussion of some of the photoconductive aspects of this problem.

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[1]A. Honig, in Proceedings of the International Conference on Semiconductor Physics, Prague, 1960 (to be published).

[2]R. C. Fletcher, W. A. Yager, G. L. Pearson, A. N. Holden, W. T. Read, and F. R. Merritt, Phys. Rev. 94, 1392 (1954).

[3]With shorter wavelengths, where electron-hole pairs are produced mostly on the illuminated surface, the traps still get filled within the bulk, even though the free-electron diffusion length is very small. The nature of the diffusion process has not yet been ascertained. It may be due to hopping from large-orbit P^- traps to empty P^+ centers, or possibly to leakage of long-wavelength radiation.

[4]W. Spitzer and H. Y. Fan, Phys. Rev. 108, 268

(1957).

[5]A. Honig, Quantum Electronics-Resonance Phenomena Conference, September, 1959 (Columbia University Press, New York, 1959), p. 450.

[6]For a purer sample (about 10^{14} P/cm^3), larger photocurrent enhancements were obtained upon filling the P^+ traps with electrons. The question arises as to whether a superior photodetector can be made with the double-illumination technique. For background-limited photodetectors, an improvement in the signal-to-noise ratio is possible when the background and signal are in different spectral regions. We would like to thank Dr. Henry Levinstein for discussion of this point.

VACANCY INTERACTIONS IN SILICON

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The production and properties of vacancies in silicon are subjects upon which much empirical work has been done. For the most part the interpretation of the data in terms of detailed models has been inconclusive. Recently the interaction of radiation-induced defects (suggested to be vacancies) with chemical impurities has been demonstrated.[1] In this Letter, experiments of a new type are described which demonstrate that vacancies do indeed interact with certain chemical impurities. These experiments involve the trapping and annihilation of vacancies by interstitial impurities which thereby become substitutional. The effect has been detected by spin resonance techniques for the impurities manganese and chromium; vacancies were introduced into silicon by the precipitation of copper or silver, or by electron irradiation. The experiments involving Cu and Mn will be described first, followed by a brief description of those involving Cu and Cr, and of the irradiation experiments.

Manganese has been previously detected in Si in the forms Mn^- and Mn^{++}. Samples have now been prepared in a manner similar to that previously described[2] except that Cu as well as Mn is alloyed and diffused into the samples. When such samples are cooled relatively slowly, two new spectra are seen due to isolated Mn, one in n-type and the other in p-type material. It is proposed that if only Mn is introduced and if the sample is quenched sufficiently rapidly, the Mn is in interstitial sites. On the other hand, in samples which also contain copper, vacancies are created during the quenching. These vacancies are annihilated by the interstitial Mn with most of the Mn becoming substitutional. In n-type silicon containing an excess of phosphorus, the substitutional Mn is in the form $Mn^=$. In p-type silicon containing an excess of boron, the substitutional Mn is in the form Mn^+.[3] A summary of the spin resonance results for the four isolated forms of Mn is given in Table I.

The resonance measurements indicate that the Mn in each of these cases is in a tetrahedral crystalline environment as an isolated impurity. The two negatively charged species of Mn are both seen in the presence of uncompensated phosphorus, indicating that they are not different charge states of one impurity site, but represent different sites. There are only two different sites which show the tetrahedral symmetry of the host lattice, the substitutional site and the interstitial site. Thus one species must be substitutional and the other interstitial. Similar arguments apply to the two positively charged species, since each is seen in silicon containing an excess of boron.

A dramatic difference between Cu-doped and Cu-free samples is the stability of the resonance center. For example, the Mn^{++} spectrum dis-

Table I. Electron spin resonance parameters for four isolated species of Mn in Si. S is the electron spin, a, the cubic field interaction parameter, and A, the (isotropic) hyperfine interaction parameter of the Mn nucleus. Both a and A are expressed in units of 10^{-4} cm^{-1}.

	Interstitial		Substitutional	
Ion	Mn^+	Mn^-	Mn^+	$Mn^=$
S	5/2	1	1	5/2
g	2.0066	2.0104	2.0259	2.0058
a	+19.88	...	...	+26.1
A	-53.47	-71.28	-63.09	-40.5

appears in a few days if samples are left at room temperature. Even when samples are stored at dry ice temperature, a decline in intensity is observed. In contrast, the Mn^{+} spectrum has been observed with little decrease in intensity in a sample left at room temperature for 18 months. The greater stability of the Mn^{+} site is strong evidence that it is the substitutional site while the Mn^{++} site is the interstitial. Consistent with this the $Mn^{=}$ site is more stable than the Mn^{-} site. Further evidence that the Mn in the Cu-free sample is interstitial is its ability to diffuse at room temperature to form clusters of four Mn atoms and to form pairs with acceptor atoms.[2] Only interstitial ions would be expected to exhibit such room temperature mobility since vacancies are normally not present. Finally, a consistent picture[4] of the d-shell configurations and electron spin values can be made on the basis of the substitutional-interstitial assignments proposed here.[5]

Although the mechanism by which the Cu creates vacancies is not certain, several interesting possibilities exist. The solubility of copper at the diffusion temperature is about $10^{18}/cm^3$, while only $5\times 10^{14}/cm^3$ has been detected electrically.[6] Presumably the rest of the copper precipitates when the sample is cooled. If a sufficient number of Cu atoms are substitutional at the diffusion temperature, vacancies may be formed by a (substitutional)→(vacancy + interstitial)→(vacancy + precipitate) mechanism as the sample is cooled. Another possibility, proposed to us by Dash, is that Cu precipitates strain the crystal, causing the formation of dislocation loops as found by Parasnis and Mitchell in AgCl.[7] The motion of these loops through the crystal then produces the needed vacancies. It is noted that silver will also convert Mn in silicon to substitutional sites. Other impurities may act similarly.

Similar experiments have been performed on Cr-doped silicon. Chromium has been previously detected[2] as an interstitial in the form Cr^{+}. If both Cr and Cu are introduced into material that initially has high resistivity, a new spectrum is seen, which is attributed to substitutional Cr^{0}. The observed spin is 1.

When silicon containing interstitial Cr is irradiated with 1.5-Mev electrons (total flux of approximately $6\times 10^{16}/cm^3$), the same center is created that is observed in silicon doped with both Cr and Cu. The samples were kept below 175°K during the irradiation and were warmed to room temperature for a short period following the irradiation (less than 30 minutes for one sample). Thus vacancies are produced by the irradiation which can be trapped by interstitial Cr. The experiments to date indicate that vacancies are mobile at room temperature and probably lower.[8] This is consistent with the results of Watkins et al.[1]

The authors thank W. C. Dash for helpful discussions and J. W. Corbett for irradiating samples. C. R. Trzaskos assisted in the experimental work.

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[1]G. D. Watkins, J. W. Corbett, and R. M. Walker, J. Appl. Phys. 30, 1198 (1959); G. Bemski, J. Appl. Phys. 30, 1195 (1959).

[2]H. H. Woodbury and G. W. Ludwig, Phys. Rev. 117, 102 (1960).

[3]The assignment of the charge state for these ions follows from the observation that neither resonant form is observed in material which had high resistivity before diffusion. This means that the difference in charge state of the Mn ion in phosphorus-doped material and boron-doped material must be at least two. However, the difference is odd since the observed spin is even for one ion and odd for the other. Thus, the charge states must differ by at least three electrons. The assigned charge states are the lowest consistent with these requirements.

[4]G. W. Ludwig and H. H. Woodbury, following Letter [Phys. Rev. Letters 5, 98 (1960)].

[5]The new observations presented in this Letter on Mn plus the fact that many of the arguments as to its interstitial vs substitutional character also apply to the impurities studied in reference 2, lead us to propose that all of the impurities discussed in reference 2 are interstitial.

[6]C. B. Collins and R. O. Carlson, Phys. Rev. 108, 1409 (1957).

[7]A. S. Parasnis and J. W. Mitchell, Phil. Mag. 4, 171 (1959).

[8]These experiments do not preclude the possibility that vacancies diffuse as di-vacancies or as some other complex.

ELECTRONIC STRUCTURE OF TRANSITION METAL IONS IN A TETRAHEDRAL LATTICE

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Although transition metal ions have been studied for many years as impurities in lattices of tetrahedral symmetry (e.g., silicon and germanium), little information has emerged as to their electronic structure in such an environment.[1] In this Letter the authors present a model for the electronic structure of ions of the $3d$ transition metals in silicon, deduced from electron spin resonance measurements. This model is the following[2]:

1. The tetrahedral crystalline field partially lifts the fivefold orbital degeneracy of the $3d$ shell. In substitutional transition metal ions the resulting threefold degenerate t_2 states lie higher in energy than the doubly degenerate e states. The reverse is true for the interstitial ions.

2. The substitutional transition metal ions transfer enough $3d$-shell electrons to the valence shell to form tetrahedral bonds with the silicon nearest neighbors. The interstitial ions transfer all valence-shell electrons to the $3d$ shell.

The effect on transition metal ions of a crystalline field of octahedral symmetry is well known.[3,4] Consider a single $3d$ electron ($l=2$). A cubic crystalline field splits the fivefold orbital degeneracy into a doublet (e_g) and a triplet (t_{2g}). The lobes of the doublet wave functions point in the cubic crystalline directions (the angular dependences are as $3z^2-r^2$ and as x^2-y^2) while the lobes of the triplet wave functions point along the face diagonals (the angular dependences are as xy, yz, and zx). For a positive ion surrounded by an octahedron of negative ions, the energy of the e_g states is higher than that of the t_{2g} states, since the lobes of the wave functions of the e_g states extend toward the negative ions. Calculations have also been made for a tetrahedral arrangement of negative ions about a central positive ion. In this case the order of the levels is reversed.[4]

Transition metal ions in silicon represent a somewhat different case, since the silicon atoms are bound covalently. Still, it is expected that for substitutional ions the e states lie lower in energy than the t_2 states, since the regions of lower electron density are in the cubic directions. Electron spin resonance measurements on substitutional Mn and Cr ions support this view.[5,6] Substitutional $Mn^=$ has an electron spin of 5/2;

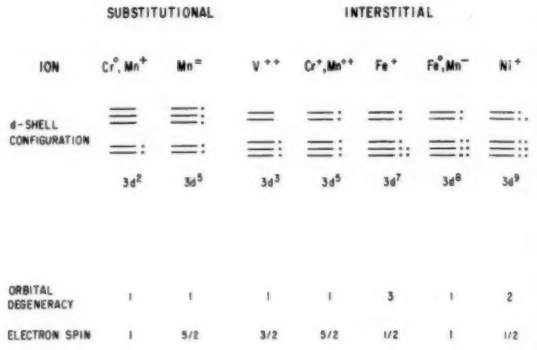

FIG. 1. Proposed electronic structure of $3d$ transition metal ions in silicon. The orbital degeneracy and electron spin values (derived from the proposed structure) agree with the spin resonance results. The spin of 1/2 for Fe^+ results from spin-orbit interaction (see text).

the proposed model is that four electrons are used to form tetrahedral bonds to the silicon nearest neighbors, leaving the configuration $3d^5$ for the manganese (see Fig. 1). The charge state of manganese can be varied by varying the concentration and type of the doping agent, thereby varying the position of the Fermi level in the crystal. If the t_2 levels are higher in energy, as assumed, and if the $Mn^=$ ion loses electrons from the d shell rather than from the bonding orbitals when its charge state is changed, then substitutional Mn^- and Mn^0 are orbital triplets. The orbital and spin degeneracy of such triplets is partially lifted by spin-orbit interaction. However, the resulting states are frequently spaced so closely that relaxation times are short even at low temperature and spin resonance is difficult to detect.[7] The authors have not been able to detect Mn^0 or Mn^- in suitably prepared crystals at temperatures as low as 10°K.

Removing a third electron would result in an orbital singlet (Mn^+). Electron spin resonance due to substitutional Mn in this charge state, and due to the isoelectronic Cr^0, has been detected at 20.4°K. The observed electron spin values and the long relaxation times substantiate

the view that the electron configuration is $3d^2$ and that the t_2 levels lie higher in energy.

The spectra of interstitial transition metal ions in silicon[5] cannot be explained by the assumptions used in the substitutional case. The results can be accounted for if the triply degenerate t_2 states lie lower in energy than the e states. A qualitative explanation for this is as follows: While the symmetry of the four nearest neighbors to an interstitial site is tetrahedral, the six second nearest neighbors, which are almost as close (the ratio of distances being $2/\sqrt{3}$), are arranged in a regular octahedron. Apparently the tendency of the second nearest neighbors to raise the energy of the e states overrides the relative lowering of these states via interaction with the nearest neighbor silicon atoms.

The second hypothesis used to account for the spin resonance observations on interstitial ions is that all valence-shell electrons are transferred to the $3d$ shell.[8] In free space, such a transfer requires about 1 to 4 ev per neutral atom, and about 0 to 1 ev per singly charged positive ion.[9]

The electronic structures assigned to the various interstitial ions in silicon are shown in Fig. 1. In the case of V^{++}, all of the valence-shell electrons are ionized and the d-shell configuration is that of the free atom. Since the t_2 states lie lower in energy, the ground state is an orbital singlet and resonance absorption near $g=2$ is observed ($S=3/2$). Resonance due to Mn^{++} and Mn^{-} is observed, since these ions are orbital singlets; again the g factors are near two, and the electron spin values are as predicted. Resonance due to Mn^{+} and Mn^{0} has not been observed at temperatures as low as 10°K; the ground state is an orbital triplet. The spectrum of Fe^{0} is similar to that of Mn^{-}, indicating that the electron configurations are identical. If the configuration of Fe^{0} were that of the atom in free space, the observed electron spin would be two rather than one.

The model predicts that the ground state of Fe^{+} in silicon is an orbital triplet (neglecting spin-orbit interaction) and that the spectrum is similar to that of Co^{++} in a crystalline field of octahedral symmetry. One indeed only detects resonance for Fe^{+} at very low temperatures ($T \lesssim 10$°K); the g factor (3.5) is similar to those found for Co^{++}. Following Abragam and Pryce,[10] it appears that this g factor (and the spin of 1/2) can be accounted for in terms of a tetrahedral crystalline field, spin-orbit interaction, and the mixing of states derived from the atomic ^4P term with those of the ^4F ground state.

The model predicts a Jahn-Teller distortion and a spin of 1/2 for Ni^{+}, and such is observed.[5]

We thus feel that the occupation of the $3d$ shell of transition metal ions in silicon varies widely depending upon whether the ion is substitutional or interstitial, and depending on its charge state. In particular, the configuration[11] of manganese ranges from $3d^2$ to $3d^8$. Contrary to what might have been anticipated, in interstitial ions the t_2 states lie lower in energy than the e states.

The authors are indebted to F. Ham for valuable discussions

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[1]See, for example, N. B. Hannay, Semiconductors (Reinhold Publishing Corporation, New York, 1959), p. 340 ff; Richard A. Bube, Photoconductivity of Solids (John Wiley and Sons, Inc., New York, 1960), p. 135 ff, 158 ff; W. Low and M. Weger, Phys. Rev. 118, 20 (1960).

[2]The notation used to describe the $3d$ states follows that of J. D. Dunitz and L. E. Orgel [J. Phys. Chem. Solids 3, 20 (1957)]. See also R. S. Mulliken, Phys. Rev. 43, 279 (1933).

[3]H. A. Bethe, Ann. Physik 3, 133 (1929); K. D. Bowers and J. Owen, Reports on Progress in Physics (The Physical Society, London, 1955), Vol. 18, p. 305; J. D. Dunitz and L. E. Orgel, J. Chem. Phys. Solids 3, 20 (1957).

[4]See, for example, W. Low, in Solid-State Physics, edited by F. Seitz and D. Turnbull (Academic Press, New York, 1960), Suppl. 2, Chap. II.

[5]H. H. Woodbury and G. W. Ludwig, Phys. Rev. 117, 102 (1960); G. W. Ludwig and H. H. Woodbury, Phys. Rev. 117, 1286 (1960); H. H. Woodbury and G. W. Ludwig, Bull. Am. Phys. Soc. 5, 158 (1960).

[6]All of the transition metal ions reported on in reference 5 are interstitial. Since that time we have been able to produce and detect manganese and chromium in a substitutional form as well. These experiments, and evidence for the substitutional or interstitial character of ions, are discussed in the preceding Letter [H. H. Woodbury and G. W. Ludwig, Phys. Rev. Letters 5, 96 (1960)].

[7]See, for example, W. Gordy, W. V. Smith, and R. F. Trambarulo, Microwave Spectroscopy (John Wiley & Sons, New York, 1953), p. 224.

[8]This hypothesis was first suggested to us by G. D. Watkins. He reasoned that the radial extent of the $4s$ orbital is greater than that of the $3d$ orbitals, and that in the interstitial site overlap with orbitals of the neighboring silicon atoms raises its energy above that of the $3d$ shell.

[9]C. E. Moore, Atomic Energy Levels, National Bureau of Standards Circular No. 467, August 15, 1952 (U. S. Government Printing Office, Washington, D. C., 1952), Vol. 2.

[10]A. Abragam and M. H. L. Pryce, Proc. Roy. Soc.

(London) A206, 173 (1951).

[11]It is noted that the 3*d* orbitals in the solid may be hybridized with other orbitals having similar transformation properties, and, as W. Marshall has pointed out to us, their radial extent may be considerably greater than that in the free atom.

INTERACTION OF PHONONS AND SPIN WAVES IN YTTRIUM IRON GARNET

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Schlömann[1] has shown that certain spin wave pairs can be made to grow in amplitude when coupled by a uniform rf magnetic field applied parallel to the dc biasing field. The strength of the rf field required to initiate unstable growth is a measure of the width, ΔH_k, of the spin wave resonance. If a given biasing field is applied to a small sample of ferrimagnetic material in a microwave cavity, measurement of the power level at which the return loss from the cavity changes discontinuously allows one to plot a curve such as Fig. 1. The branch labelled $\theta = \pi/2$ is of particular interest since it corresponds to spin waves of known wavelength propagating in a particular direction. The linewidth of these spin waves is given by[1] $4\pi\gamma M h_{rf}/\omega_p = \Delta H_k$, where ω_p is the pumping frequency and the wave number, k, of the $\omega_p/2$ spin waves is specified by[2]

$$Dk^2/\gamma\hbar = H_m - H. \qquad (1)$$

Here H is the applied dc field and H_m is the limiting value of H for $\frac{1}{2}\pi$-directed spin waves as k approaches zero. Since the ΔH_k of specific spin waves is determined by this experiment, one can check the adequacy of the Kasuya, Sparks, and Kittel[3,4] linewidth theory and measure D at a higher temperature than is possible with other techniques.[5-7] This experiment is still in progress and will be reported on jointly with E. G. Spencer and R. C. LeCraw later.

The magnitude of ΔH_k is shown in Fig. 2 as a function of $(H_m - H)^{1/2}$ for a 0.019-inch yttrium

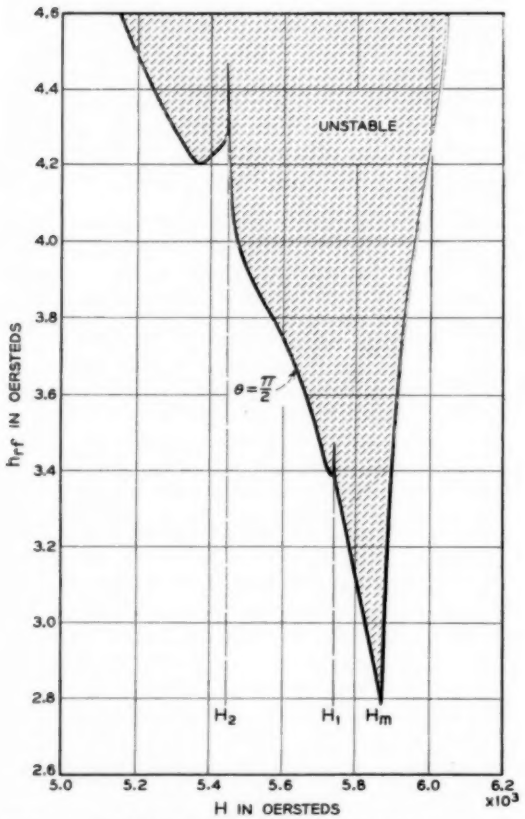

FIG. 1. Rf field strength required to excite spin wave pairs as a function of applied field. Pumping frequency is 34.627 kMc/sec, and fields are along [111] crystal axis.

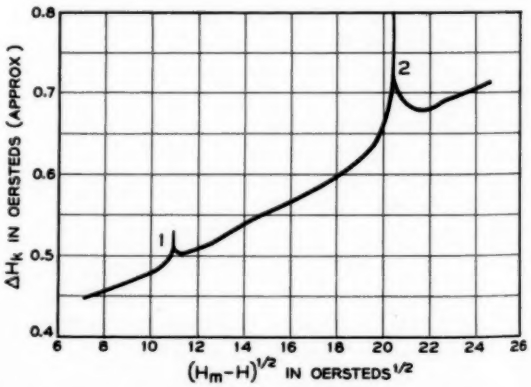

FIG. 2. ΔH_k as a function of $(H_m - H)^{1/2}$. The anomalies are indicated at 1 and 2.

iron garnet (YIG) sphere at 300°K and with a pumping frequency 34.627 kMc/sec. According to the dispersion relation (1), the abscissa should be proportional to k. The value of H_m used is the experimentally observed value. Fortunately, this agrees within experimental error with the theoretical value of H for $\frac{1}{2}\pi$-directed spin waves with $k=0$, so one has some confidence in treating H_m as a known rather than an adjustable parameter. The sharp peaks in ΔH_k at H_1 and H_2 are believed to be caused by coupling of spin waves to phonons[8] of the same frequency ($\omega_p/2$) and the same $\vec{k}$. A 4.5% reduction in pumping frequency reduced $(H_m - H_{1,2})^{1/2}$ by 4.5%, as it should if the phonon dispersion relation is $\omega_S = vk_S$.

Further evidence on the frequency dependence has been obtained by LeCraw and Spencer who performed the same experiment at 11.4-kMc/sec and 9.2-kMc/sec pump frequencies and found the anomalies in ΔH_k at very nearly the predicted values of H. The deviation from the predicted value indicates that $(\partial\omega/\partial k)$ has decreased slightly at 17 kMc/sec.

The peak in ΔH_k observed at H_2 is seen to be considerably larger than that at H_1. This may be caused by a difference in the nature of the phonons involved in the two peaks or it may be simply that higher k waves are more strongly coupled. The latter argument is supported by the fact that the interaction is considerably weaker at lower frequencies. Examination of the curves near the peak at H_2 shows some distortion associated with the peak, indicating the need to deal with the coupled system of magneto-acoustic waves. The apparent sharpness of the interaction is probably due to the fact that $(\partial\omega/\partial k)$ is very different for the acoustic waves and spin waves at the crossover region and this, rather than the Q of the acoustic waves, is important. The large attenuation constant of the acoustic waves at room temperature primarily reduces the magnitude of the interaction.

Data on the elastic constants of single-crystal YIG are not yet available, but with certain simple assumptions one can use McSkimin's values of sound velocity in polycrystalline material[9]: $v_t = 3.87\times10^5$ cm sec^{-1}, $v_l = 7.17\times10^5$ cm sec^{-1}. If YIG is elastically isotropic, then one can assign the peak at H_1 to longitudinal phonons and the peak at H_2 to transverse phonons since the ratio $(H_m - H_2)^{1/2}/(H_m - H_1)^{1/2} = 1.86$ and $v_l/v_t = 1.85$. Using the relation (1), $D = 0.94\times10^{-28}$ erg cm^{-2} from 11.4-kMc/sec data, or 0.99×10^{-28} erg cm^{-2} from 34.6-kMc/sec data at room temperature. On the other hand, if YIG follows the Cauchy relation for a cubic crystal and we assign both peaks to transverse waves with $v_{t_1} = [(C_{11} - C_{44})/\rho]^{1/2}$ and $v_{t_2} = (C_{44}/\rho)^{1/2}$, then $D = 0.45\times10^{-28}$ erg cm^{-2} from 34.6-kMc/sec data. These values of D are to be compared with low-temperature specific heat[5-7] values of 0.55×10^{-28} to 0.86×10^{-28} erg cm^{-2}.

Assuming that the observed effects are due to phonons, measurement of the elastic constants of YIG will permit a determination of D as a function of temperature by this method and the magneto-acoustic interaction can be studied rather directly.

It is a pleasure to acknowledge helpful conversations with several colleagues, including I. P. Kaminow, L. R. Walker, and particularly R. C. LeCraw. The author is much indebted to F. Dacey who performed many of the experiments.

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THEORY OF A HEISENBERG FERROMAGNET IN THE RANDOM PHASE APPROXIMATION*

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The aim of this Letter is to present a solution of the ferromagnetic problem in the Heisenberg model in random phase approximation (RPA). This is conjectured to be the high-density limit theory. In this preliminary account we restrict ourselves to spin one-half and to a study of magnetic properties alone.

The principal result of the analysis is a magnetization curve which imitates conventional spin wave theory very closely but with the existence of renormalized spin wave frequencies according to $\omega(\vec{q};T)=\omega(\vec{q};0)M(T)/M(0)$, where $M(T)$ is the magnetization at temperature T, in accordance with an idea of Brout.[1] The theory goes over to the spherical model[2] at temperature higher than the Curie temperature for small static fields with the Curie point given by the spherical model value. Thus we have a consistent extrapolation formula through the whole temperature range possessing rigorously correct limiting properties for $T\to 0$ and $T\to\infty$, as well as a presumably accurate value of the Curie temperature in the limit of high density.

We write the Heisenberg Hamiltonian in the form

$$\mathcal{H}=-2\sum_{\vec{q}}v(\vec{q})\{S^z(\vec{q})S^z(-\vec{q}) + \tfrac{1}{2}[S^+(\vec{q})S^-(-\vec{q})+S^-(\vec{q})S^+(-\vec{q})]\}, \quad (1)$$

where we have Fourier-analyzed the exchange potential $v(\vec{r})$ and the spin operators $S(\vec{q})=N^{-1/2}\sum_i S_i\exp(i\vec{q}\cdot\vec{r}_i)$. The random phase approximation can be summarized by the simplified commutation rule which expresses the decoupling of different Fourier components:

$$[S^+(\vec{q}),S^-(\vec{q}')]_-=\delta_{\vec{q},-\vec{q}'}(2/\sqrt{N})S^z(0),$$
$$[S^z(\vec{q}),S^\pm(\vec{q}')]_-=\pm\delta_{\vec{q},0}(1/\sqrt{N})S^\pm(\vec{q}'), \quad (2)$$

where N is the number of spins.

We apply to the system a finite longitudinal magnetic field H and switch on adiabatically an infinitesimal transverse rotating magnetic field of wave vector $\vec{q}'$. This adds to the Hamiltonian (1) a term,

$$\mathcal{H}'=-S^+(-\vec{q}')h_{\vec{q}'}^{-}e^{i(\omega-i\epsilon)t}-g\mu\textstyle\sum_i S_i^z H. \quad (3)$$

Denoting by $\langle O\rangle_{T,H}$ the thermal average at temperature T and external magnetic field H of the operator O, we may define the adiabatic spin susceptibility,

$$\chi_{\vec{q}'}^{-}(\omega)_{T,H}=\lim_{h_{\vec{q}'}^{-}\to 0}\left(\frac{\langle S^-(\vec{q}')\rangle_{T,H}}{h_{\vec{q}'}^{-}e^{i(\omega-i\epsilon)t}}\right). \quad (4)$$

Writing the Heisenberg equations of motion, one easily finds with help of (2),

$$\chi_{\vec{q}'}^{-}(\omega)_{T,H}=\frac{R}{[v(0)-v(\vec{q})]2R+g\mu H-\hbar\omega+i\epsilon}, \quad (5)$$

where R is the reduced magnetization, $R=2N^{-1}\times\langle\sum_i S_i^z\rangle_{T,H}$. From the Kubo formalism,[3] the appropriate form of the fluctuation-dissipation theorem is easily derived[4]:

$$\int_{-\infty}^{\infty}\frac{1}{1-\exp(\hbar\omega/kT)}\,\mathrm{Im}\chi_{\vec{q}}^{-}(\omega)_{T,H}d\omega = (i\pi/\hbar)\langle S^-(\vec{q})S^+(-\vec{q})\rangle_{T,H}, \quad (6)$$

where χ in Eq. (6) is the adiabatic susceptibility, which in RPA is given by Eq. (5). Substituting Eq. (5) into Eq. (6), we have

$$\frac{R}{\exp\{[(v(0)-v(\vec{q}))2R+g\mu H]/kT\}-1}=\langle S^-(\vec{q})S^+(-\vec{q})\rangle_{T,H}. \quad (7)$$

In a completely similar way, we obtain

$$\frac{R}{1-\exp\{-[(v(0)-v(\vec{q}))2R+g\mu H]/kT\}}=\langle S^+(\vec{q})S^-(-\vec{q})\rangle_{T,H}. \quad (8)$$

Using the sum rule,

$$\textstyle\sum_{\vec{q}}[S^+(\vec{q})S^-(-\vec{q})+S^-(\vec{q})S^+(-\vec{q})]=N, \quad (9)$$

we obtain the following equation for the magneti-

zation:

$$N = R\sum_{\vec{q}} \coth\left(\frac{[v(0) - v(\vec{q})]2R + g\mu H}{2kT}\right). \quad (10)$$

Equation (10) has two classes of solutions in the limit $H \to 0$.

(a) $H \to 0$, $R \neq 0$, or the ferromagnetic region.

The physical interpretation of (10) in the ferromagnetic region is best seen by writing (10) (as $H \to 0$) in "spin wave" form:

$$n = \sum_{\vec{q}} \frac{R}{\exp\{[v(0) - v(\vec{q})]2R/kT\} - 1}, \quad (11)$$

where n is the number of "spin deviations"; $n = \frac{1}{2}N(1 - R)$. Thus the magnetization curve (11) appears as a consequence of the existence of collective spin motions generalizing at finite temperature the well-known spin waves by renormalizing their frequency by a factor R. This concept of renormalized spin waves, which leads to a T^3 correction to the usual spin-wave theory for the magnetization curve at low temperature, seems to be in contradiction with Dyson's theory,[5] and this discrepancy will be studied in detail in the future. This concept and the T^3 law which results has been already introduced by Brout and Haken[1] from a physical point of view. The magnetization curve obtained in reference 1, however, is not the same as the present one. A detailed study has been made of this discrepancy by Brout. It has turned out that the reasoning in reference 1 contained an inconsistency in the evaluation of traces in RPA. In a subsequent publication it will be shown how the final result [Eq. (10)] can be obtained by a consistent RPA in the partition function.

The Curie point determined from (10) or (11) is obtained by taking the limit $R \to 0$, $H \to 0$:

$$1/kT_c = F(1)/v(0), \quad (12)$$

with

$$F(1) = \frac{1}{N}\sum_{\vec{q}} \frac{1}{1 - [v(\vec{q})/v(0)]},$$

which is the Curie point of the spherical model.[2]

(b) $H \to 0$, $R \to 0$, or the paramagnetic region.

The susceptibility being defined by $\chi = \frac{1}{2}g\mu R/H$ as $H \to 0$, one finds immediately from (10)

$$1 = \frac{1}{N}\sum_{\vec{q}} \frac{1}{\beta[v(0) - v(\vec{q})] + \beta(g\mu/2)^2/\chi}. \quad (13)$$

If we write

$$\chi = \left(\frac{g\mu}{2}\right)^2 \beta \frac{1}{1 - \beta[v(0) - \delta]}, \quad (14)$$

one sees from (13) that δ is determined by

$$1 = \frac{1}{N}\sum_{\vec{q}} \frac{1}{1 - \beta[v(\vec{q}) - \delta]}, \quad (15)$$

which shows the equivalence of our paramagnetic curve with that obtained from the spherical model.[2] Because of the work of Brout,[6] we speculate on the present theory as the high-density limit theory.

A complete discussion of these results, as well as the thermodynamic and nonequilibrium properties, will be published in a forthcoming paper.

We should like to thank R. Brout for his help in this study as well as for many very interesting discussions.

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†On leave of absence from Université Libre de Bruxelles, Bruxelles, Belgium.

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THERMAL PROPERTIES OF SOLID He⁴

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(Received July 7, 1960)

The predicted[1] anomalous thermal properties of solid He^3, in the absence of qualitative modifications of the nuclear paramagnetism of the anomalous liquid in equilibrium with it, along the phase separation line, raised the following problem. Do the anomalous thermal properties of liquid He^4 persist also in the solid at, or near, the melting line or not? Results obtained in the study of this problem are presented here, a detailed account being reserved for a later paper.

The anomalous thermal properties of liquid He^4 occur over that area of the state surface, which projected on the pressure-temperature, (p, T), plane is enclosed by the lines $p_0(T)$, $p_m(T)$, $T_\alpha(p)_{II}$, and $T_\alpha(p)_I$. These are, respectively, the saturation and melting pressure lines, and the loci of the temperatures of vanishing isobaric volume expansion coefficients. Empirical information[2] is fairly complete on the first two, it is only fragmentary on the loci,[3] with $T_\alpha(p)_I$ being a line in the He^4 I region, quite close to the locus of lambda points, $T_\lambda(p)$. The following relations describe the anomalous thermal properties of liquid He^4 at or near the melting line, V and S standing for its volume and entropy:

$$\alpha_p(p_m)=[V(p_m)]^{-1}(\partial V/\partial T)_p\leq 0;\quad [\partial S(p_m)/\partial p]_T\geq 0;$$
$$(\partial T/\partial p)_S\leq 0;\quad T_\alpha(p_m)_{II}\leq T\leq T_\alpha(p_m)_I. \qquad (1)$$

We have to invoke now the experimental rule, of thermodynamic character,

$$\alpha_{p,l}(p_m)\geq\alpha_{p,s}(p_m), \qquad (2)$$

which states that, in normal substances, the isobaric volume expansion coefficent $\alpha_{p,l}$ of the liquid, in equilibrium with the solid along the $p_m(T)$ line, is larger than, or at least equal to, that of the solid, $\alpha_{p,s}$. If the liquid is anomalous, as described by (1), one proves that (2) must be generalized to

$$|\alpha_{p,l}(p_m)|\geq|\alpha_{p,s}(p_m)|. \qquad (3)$$

One also proves that equalities in (2) and (3) are allowed only at isolated p_m values, where the α_p's vanish. In the He isotopes, this occurs, by the Nernst theorem, at the absolute zero. Relations (1) and (3) lead unambiguously to two possible thermal behaviors of solid He^4:

I. In this case of least anomaly, the solid has normal thermal properties with the exception of the isolated states $T_\alpha(p_m)_{II}$ and $T_\alpha(p_m)_I$, where $\alpha_{p,s}(p_m)$ vanishes. This property must have at least one maximum below $T_\alpha(p_m)_{II}$ and one between it and $T_\alpha(p_m)_I$. The solid entropy $S_s(p)$ must have pressure maxima at the zeros of $\alpha_{p,s}$.

II. Alternatively, solid He^4 along or near $p_m(T)$ must be completely anomalous, with the set of properties (1), with zeros of $\alpha_{p,s}$ at $T_\alpha(p_m)_{II}$ and $T_\alpha(p_m)_I$.

Analysis of experimental data available to date[2] on liquid and solid He^4, along $p_m(T)$, at $T\geq 1.1$ - $1.2°K$, yields, through the rigorous formula,

$$\alpha_{p,s}(p_m)=[V_s(p_m)]^{-1}(dV_s/dT)$$
$$+\kappa_{T,s}(p_m)(dp_m/dT), \qquad (4)$$

$\kappa_{T,s}$ being the isothermal compressibility, negative $\alpha_{p,s}$ values below about 1.50 - 1.55°K. Only upper limits of $\alpha_{p,s}(p_m)$ could, however, be obtained because only approximate upper limits of $\kappa_{T,s}$ were available. The actual intermediate zero of $\alpha_{p,s}(p_m)$, if below $T_\alpha(p_m)_I$, must be higher than 1.55°K. Accordingly, solid He^4 is completely anomalous over, at least, a limited range of the interval $[T_\alpha(p_m)_I - T_\alpha(p_m)_{II}]$ and, probably, over a pressure range above $p_m(T)$.

These results lend support to the idea developed[1] in connection with He^3, and verified experimentally,[4] that quantum effects, together with the pertinent statistics,[5] responsible for the thermal anomalies of the liquid isotopes, persist over limited regions of the solid phases of the He isotopes. The role played by the "normal" fluid in the solidification clearly emerges in these studies. The existence of a shallow minimum of $p_m(T)$, at $T\leq T_\alpha(p_m)_{II}$ (~1.0°K) is predicted if the dominant thermal excitations of the solid are phonons at low temperatures.

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REGULARITIES OF (d, α) REACTIONS IN HEAVY ELEMENTS*

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We are presenting the preliminary results of a survey of (d, α) reactions in heavy elements with an incident deuteron energy of 15 Mev. A proportional counter-scintillator telescope was used in conjunction with a 256-channel analyzer to obtain energy distributions with a precision of 0.3 Mev and resolution varying between 4 and 8%.

The resulting energy distribution of alphas emitted at 90° (Fig. 1) shows two strong groups which vary regularly with atomic number. Data taken at 120°, 60°, and 30° indicate similar structure; although relative intensities change, the peaks show little or no energy shift with angle. The differential cross section at 90° falls off rapidly by a factor of 30 between $_{29}$Cu and $_{52}$Te. This decrease is associated with the rapid disappearance of the low-energy group around $Z = 50$. At this point the distribution consists predominantly of the high-energy peak which has a relatively constant cross section (~0.25 mb/sr) for increasing atomic weight. Positions of both high- and low-energy groups have been compared with known ground-state transition energies in Fig. 2. From this plot it is evident, from the separation of the ground states and the experimentally determined high-energy peaks, that the ground states are weakly excited for all nuclei investigated. In addition, there seems to exist no correlation between the ground states and the high-energy peaks. This rules out the possibility that the peaks are due to the excitation of collective states, as the latter occur at excitation energies which vary slowly with mass number.

An outstanding feature of the high-energy group is the slow regular variation of peak energy with atomic number. This effect is most apparent in the transition region from $_{45}$Rh to $_{50}$Sn, where the peak shifts by less than 0.5 Mev. Supplementary data taken on the seven most abundant isotopes of Sn also show the high-energy peak in the same position.

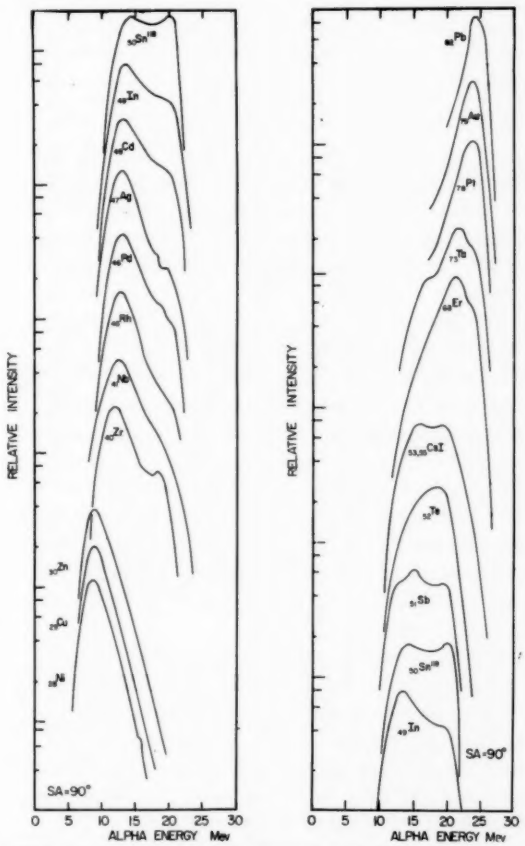

FIG. 1. Energy distributions for alpha particles emitted at 90°.

Variations in the energy distributions as a function of scattering angle are shown for $_{79}$Au,

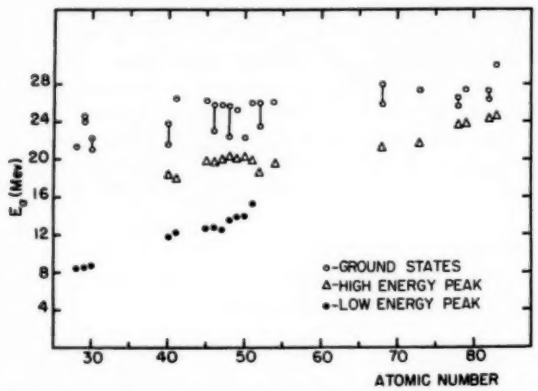

FIG. 2. Comparison of peak energies with ground-state transitions as a function of atomic number. The ground-state positions for both even and odd isotopes have been tabulated and connected with a line for the sake of clarity.

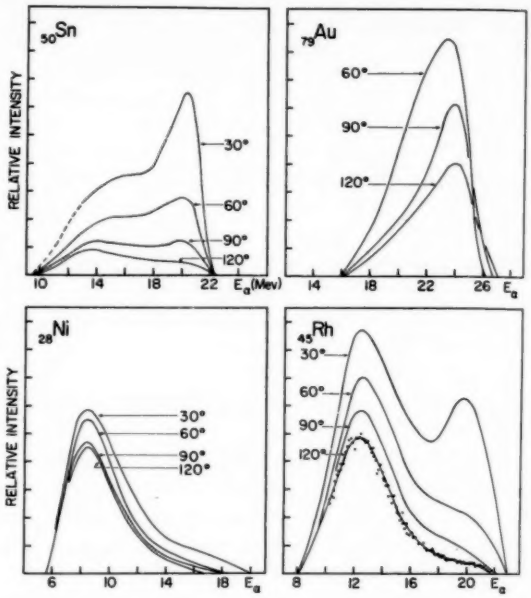

FIG. 3. Dependence of the energy distributions on scattering angle for four typical elements. These nuclei have been selected to represent trends in different regions of the periodic table.

$_{50}$Sn, $_{45}$Rh, and $_{29}$Ni in Fig. 3. This plot shows the lack of shifting in the peak energy with angle, and also shows the strong forward rise in the angular distribution for the high-energy group, an effect usually associated with direct interactions.

The energy distribution, especially in the region of Au and Pt, is characterized by a sharply peaked maximum which falls rapidly to zero on the high-energy side and somewhat more slowly with decreasing energy. These peaks, although quite narrow (~5 Mev), appear significantly wider than the resolution of the equipment (~4-5%), indicating the excitation of a group of states. When the intensity is divided by the Coulomb barrier penetration factor, there is still a strong peak; this indicates that the low-energy falloff is not due to a Coulomb cutoff.

The low-energy group in the region of $_{29}$Cu occurs at about 8.5 Mev (for $_{45}$Rh, 12.5 Mev) and shows relatively weak forward peaking in the angular distribution. The peak energies and angular distributions agree with previous results for (p, α) and (α, α') scattering,[1-3] and can be explained by compound nucleus theory. The energy distributions yield a nuclear temperature of about 1.2 Mev. The variations in the cross sections with ground-state Q values also lend support to the idea that the low-energy group consists mainly of alpha particles evaporating from a compound nucleus. This is especially evident in the even isotopes of Sn where the cross section falls by a factor of 30, in agreement with predictions from compound nucleus theory.

It seems plausible to attempt to explain the behavior of the high-energy group on the basis of either a pickup or a knockout process. The latter theory however leads to several difficulties:

1. A knockout process would be expected to result in a strong "smearing out" of the energy distribution due to internal motions of the nucleons involved.[4] The rather sharply peaked distributions, especially in the region of Au, are in conflict with this idea.

2. Assuming the existence of a sharp peak, one would expect a variation in the peak energy with angle, from a classical viewpoint. Such a variation is not observed in the data.

3. In all cases the outgoing alpha particles have a significantly greater energy than the incoming deuterons, which again conflicts with the idea of knockout.

High-energy distributions of the same shape and peaked at similar energies have been found in (p, α) work.[1] There exists strong evidence for a pickup process in (p, t) and (p, α) reactions in heavy elements[1,5,6] and also in light nuclei.[7]

A pickup mechanism can readily explain two of the most notable features of the experimental

results, the narrowness of the peak and the slow variation of its energy with mass number. The neutron and proton would each be most easily "picked up" from the single-particle states in the major shell that is filling; the single-particle states in a given major shell are closely grouped in energy, and this energy depends only on the size of the "well" so that it varies slowly with mass number. This effect has been successfully used to explain a similar regular structure in (d, p) reactions.[8] There are several difficulties with this explanation, however. The energies expected from this model are somewhat closer to those of the ground-state transitions (in fact, ground-state transitions should be relatively prominent). The shift in energy with mass number should be in the opposite direction from that observed since the levels should sink deeper into the well as the radius of the well increases. There should be sharp discontinuities at closed shells, whereas none are observed between Zr and Nb, between Sn and Sb and between Pb and Bi. It is not even clear why the energy should not exhibit even-odd energy differences similar to those of the Q values.

Most of these difficulties could be avoided if one uses the cluster model and considers a deuteron to be picked up from a (deuteron) cluster state. The shift in energy with mass number is still in the opposite direction from that expected, but this could be explained by a variation in well depth with mass number. However, it must be recognized that the apparent reduction of difficulty obtained by using the cluster model is principally due to our almost complete ignorance of the parameters of the model.

It thus seems difficult to explain the observations with either a "knockout" or a "pickup" model. Further experiments to investigate these reactions are in progress.

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MOMENTUM DISTRIBUTION OF PROTONS IN INDIVIDUAL NUCLEAR SHELLS

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We have measured the angular correlation in quasi-free p-p scattering in Li^7, and interpret the results directly in terms of the momentum distributions of the $p_{3/2}$ and $s_{1/2}$ protons, respectively.

Light nuclei are appreciably transparent to protons above 50 Mev or so, and the wavelengths of such protons are comparable with internucleon distances. If the two emergent protons from a $(p, 2p)$ reaction are required to share most of the energy of the incident proton and are detected approximately 90° apart ("quasi-free" p-p scattering), collective effects should be relatively suppressed, and there should be a strong dependence of the angular and energy correlations between the emergent protons on the momentum distribution of the struck proton. That the refraction by the collective nuclear potential does not destroy this connection has been shown by Maris[1] for the case of Li^7 and a bombarding energy of 180 Mev.

For this nucleus and this energy, earlier experiments have shown that the spectrum of the sum of the energies of two protons emerging $\theta = 90°$ apart with equal energies and at equal angles exhibits two clear peaks, one near the incident energy minus the separation energy (10 Mev) and the other 16 Mev lower.[2] We as-

cribe the first peak to the $p_{3/2}$ proton and the second to the two $s_{1/2}$ protons in Li^7.

The variation in the areas of these peaks with the opening angle should depend on the component of the momentum of the struck proton along the line of bombardment. In particular, since for free protons at rest $\theta = 90°$, the lithium s proton peak should be highest near 90°, because the most probable momentum of an s proton is zero. On the other hand, momenta near zero are improbable for p protons, and the corresponding peak should show a minimum near 90°. In both cases, the angle of the extremum should be reduced slightly by the effects of binding, and affected in a less obvious manner by refraction. The predictions of a simple model are shown in Fig. 1 of reference 1.

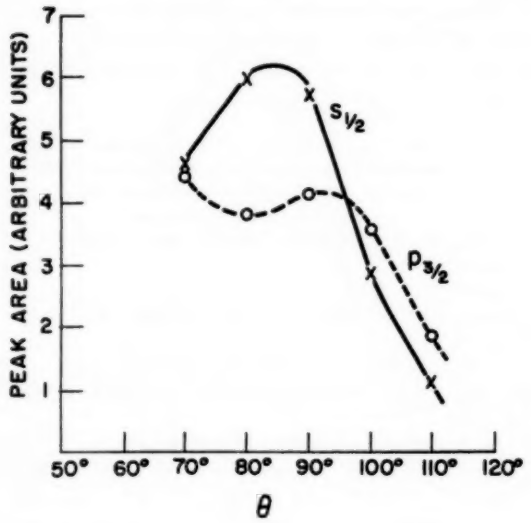

FIG. 1. The angular correlation of protons from the $(p, 2p)$ reaction on Li^7 bombarded with 180-Mev protons. The protons emerge at equal angles and with equal energies. The "$p_{3/2}$" curve corresponds to proton pairs the sum of whose energies is near 170 Mev, the bombarding energy minus the separation energy; the "$s_{1/2}$" curve to pairs with energies around 16 Mev lower. The summed-energy spectrum at each angle shows clear peaks near these energies, and the ordinate is the peak area. The abscissa is the angle between the two protons. The angular resolution is approximately a triangle with a 20° base.

To test the model, a measurement has been made in the case described above. The conditions were identical with those of reference 2 except for the variation in θ. The results are shown in Fig. 1. The behavior of the curves clearly confirms the qualitative considerations given above. Further improved experiments are in progress with other nuclei and higher bombarding energies (450 Mev), where collective effects should be smaller, and a quantitative calculation of the momentum distributions of nucleons in individual nuclear shells may then be feasible.

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$T = 0$ K^+-NUCLEON PHASE SHIFTS BASED ON THE OPTICAL MODEL*

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Using first order multiple scattering theory in the high-energy limit, Lipperheide and Saxon[1] have recently shown that the forward scattering amplitude can be immediately related to the volume integral of the optical model potential. This result has two desirable consequences for experimental situations where an attempt is made to deduce the forward scattering amplitude of the elementary scattering process from interactions with complex nuclei: (a) it clarifies the somewhat ambiguous relation between the central potential and the "central nucleon density," and (b) volume integrals of the optical model potentials are less sensitive to the shape of the potential than the central potentials themselves.

Values of the central potential, $V_0 + iW_0$, of a Saxon well, obtained by an optical model calculation designed to fit the reaction cross sections and the angular distributions for elastic scattering of K^+ mesons in emulsion at 125 Mev and at

260 Mev, have been given previously[2,3] for various assumed nuclear radius parameters R_0 ($R = R_0 A^{1/3}$) and falloff parameters a. However, the significant quantities are the volume integrals of the potentials divided by A which, for a Saxon well, are

$$I_V + iI_W = (4\pi/3)R_0^3(V_0 + iW_0)[1 + 9.88(a/R)^2]. \quad (1)$$

As an example, the variation of I_W with R_0 and a is shown in Fig. 1 for 260 Mev. These curves were determined by means of Eq. (1) and the values of V_0 and W_0 which best represented[4] the experimental data at 260 Mev for selected values of R_0 and a. The shaded area, ranging from $R_0 = 1.07$ f, $a = 0.57$ f obtained from charge density experiments[5] to $R_0 = 1.35$ f, $a = 0.85$ f which exceed the values obtained from nucleon-nucleus scattering,[6] will be regarded as a "reasonable range" of the shape parameters. In Table I we give I_V and I_W for $R_0 = 1.20$ f, $a = 0.57$ f. The uncertainties in these quantities due to this "reasonable parameter range" are given outside the parentheses and are comparable to, or smaller than, the "experimental" errors which are given inside the parentheses. The latter were obtained by finding values of V_0 and W_0 which reduced the χ^2 probability of the fit to the experimental data by a factor of three.

In the high-energy limit,[1]

$$I_V + iI_W = -(vh/k)\langle f(0)\rangle, \quad (2)$$

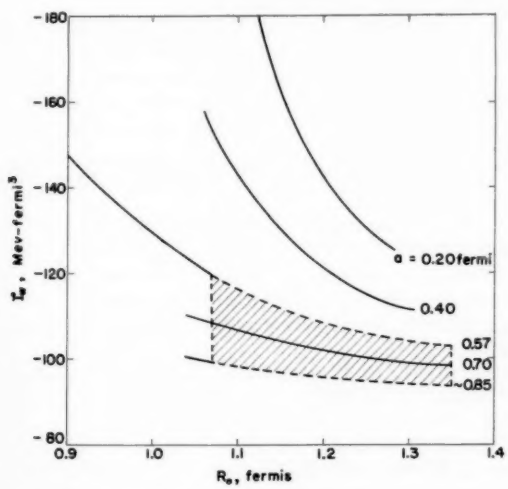

FIG. 1. Volume integral of the imaginary potential for a best fit to the 260-Mev data as a function of radius parameter R_0 and surface thickness parameter a.

Table I. Volume integrals in Mev-fermi^3 of the K^+ nuclear potentials divided by A. The uncertainties within the brackets are statistical and the deviations outside the brackets represent extreme variations of the best-fit values over a broad range of a and R_0.

T_K (Mev)	I_V	I_W
125	$(123 \pm 26) \pm 8$	$(56 \pm 8) \pm 10$
260	$(123 \pm 44) \pm 10$	$(103 \pm 13) \pm 14$

with v and k the laboratory velocity and center-of-mass wave number, respectively, of the K^+ meson. The forward scattering amplitude $\langle f(0)\rangle$ in the K-meson–nucleon center-of-mass system is averaged over protons and neutrons. In order that (2) apply at our finite energies, corrections should be applied to the observed $I_V + iI_W$. The correlation correction to I_W has been estimated to be $< 10\%$ at 125 Mev and even less at 260 Mev and has been neglected. I_W has been increased by 20% at 125 Mev and 10% at 260 Mev to take into account the effect of the exclusion principle.[7]

In the K^+-meson–nucleon phase shift analysis given below, we have used the optical model results given in Table I together with the fraction f of charge exchanges among all inelastic events at the two energies. The experimental[2,8] fraction f, the fraction F corrected for double scattering,[9] and finally the fraction ϕ corrected for the different effects of the exclusion principle on the charge-exchange and non-charge-exchange cross sections[7] are given in Table II.

We have used these results to obtain $T = 0$ phase shifts by means of a χ^2 fit under the following assumptions: (a) that due to the Coulomb and nuclear potentials, the K^+ meson energies of 125 Mev and 260 Mev are reduced to 93 Mev and

Table II. Charge exchange fractions: the experimental fraction f, the fraction F corrected for double scattering, and the fraction ϕ corrected for exclusion principle effects as well.

T_K (Mev)	f	F	ϕ
125	0.17 ± 0.03	0.15 ± 0.03	0.16 ± 0.03
260	0.32 ± 0.03	0.27 ± 0.05	0.28 ± 0.05

230 Mev, respectively; (b) that only S-wave scattering contributes in the $T=1$ state and that the δ_{10} phase shifts which correspond to a constant experimental cross section of 15 mb, are -20.6° and -34° at 93 Mev and 230 Mev, respectively[10]; and (c) that only the S- and P-wave phase shifts are appreciable in the $T=0$ state and that their momentum dependence is given by

$$\tan\delta_{00} = -a_{00}k, \quad \tan\delta_{01} = -a_{01}{}^3k^3, \quad \tan\delta_{03} = -a_{03}{}^3k^3. \tag{3}$$

Two solutions were found. These are given in Table III along with the probability P of a worse fit to the data. The uncertainties in these solutions are best shown by Fig. 2 which gives orthogonal projection of the surfaces of constant χ^2 in the a_{00}, $a_{01}{}^3$, $a_{03}{}^3$ space. The attractive P-wave nature of these solutions is in agreement with a less detailed analysis by Keefe et al.[11].

Our results disagree markedly with those of Rodberg and Thaler,[12] in spite of the fact that their analysis is based on the data of Zorn which agree well with our data and data from other laboratories[11] in those energy regions where they overlap. The discrepancy is readily traced to the difference between the diffuse surface optical model used by us and the square well used by Rodberg and Thaler. As may be seen in Fig. 1, small values of a result in an I_W (which corresponds to their λ) which is both large and strongly R_0 dependent. For example, a change of R_0 from 1.20 f, used by them, to 1.13 f would change their results by roughly their quoted uncertainties in the direction of even worse agreement. The difficulties of the square well model seem to be due to the fact that for high-energy K^+ mesons the reaction cross section σ_R approaches the geometric value.

In conclusion our results indicate that if a normal momentum dependence is assumed, the $T=0$ S-wave phase shift is small and likely to be repulsive and that one of the $T=0$ P-wave phase shifts is fairly large and attractive and the other small.

Table III. The parameters a_{00}, $a_{01}{}^3$, and $a_{03}{}^3$ in fermi and fermi^3.

Solution	a_{00}	$a_{01}{}^3$	$a_{03}{}^3$	P
A	0.07	-0.21	-0.01	0.82
B	0.09	0.05	-0.13	0.61

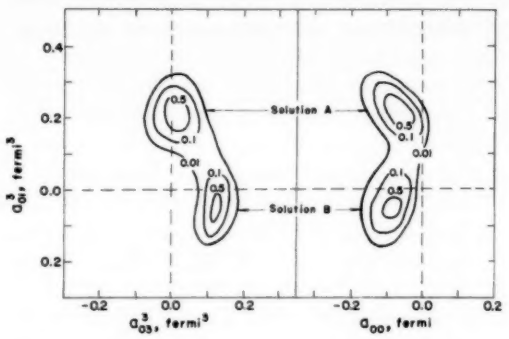

FIG. 2. Orthogonal projections of surfaces of constant χ^2 probability on the $a_{01}{}^3$-$a_{03}{}^3$ plane and on the $a_{01}{}^3$-a_{00} plane. The curves are designated by the probability value.

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[9]At $T_K = 125$ Mev, a detailed Monte Carlo method was used to find this correction as a function of the phase shifts and radius, and a typical corrected value is given. At $T_K = 260$ Mev a single correction factor was used based on the assumption that at the mean energy after the first scattering, the probability of a

second scattering with charge exchange is 0.10 ± 0.05.

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p-p PHASE SHIFT SOLUTIONS AND DEPOLARIZATION SCATTERING PARAMETER AT 210 Mev*

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Recently MacGregor and Moravcsik[1] have applied their modified phase shift analysis[2,3] to the Rochester proton-proton scattering data: cross section σ, polarization P, and the triple scattering parameters[4] A and R at 210 Mev.[5] Of their four sets of phase shifts (a through d) with lowest values of χ^2, they have excluded two (a and d) primarily on the ground that neither corresponds to one of the two acceptable phase shift solutions at 310 Mev.[3]

The Rochester nucleon-nucleon scattering program at 210 Mev has been extended to include measurement of the depolarization parameter,[4] $D(\theta)$, in p-p scattering. The preliminary data, which have been obtained thus far at 30° and 60° in the center-of-mass system, are of interest in that they definitely exclude solutions a and d of the MacGregor-Moravcsik analysis, and further indicate some preference for solution b over solution c. In addition, the data indicate an overall energy dependence for $D(\theta)$ consistent with the Harvard measurement at 150 Mev.[6,7]

The 90% polarized proton beam was scattered from a liquid hydrogen second target at an angle θ in the plane of the first polarizing scattering. The polarization perpendicular to the scattering plane for protons which scattered right and left from the second target was measured with the polarimeter[8] which employed a carbon third target and a pair of triple counter telescopes.

The twice scattered beam was defined by the illuminated volume of the hydrogen target and by a slit placed just before the third target. The polarimeter was aligned along an axis defined by two points in space: (1) the geometrical center of the slit, and (2) the center of gravity of the twice scattered beam at a distance of 35 in. beyond the slit. The uncertainty in the alignment was estimated to correspond to ± 0.003 in asymmetry, while the minimum asymmetry observed in the measurement of $D(\theta)$ was 0.110. Additional sources of spurious asymmetry resulted from the nonuniform illumination of the third target and the beam energy variation across it due to the angular dependence of the p-p scattering cross section and the kinematics. To estimate these effects, the polarimeter was calibrated with the degraded beam which passed through a wedge absorber and a collimator before striking the third target. The absorber and the collimator produced the energy and intensity variations that were expected in the hydrogen scattered beam. Each of these two effects gave a spurious asymmetry of 0.005 ± 0.006.

After making corrections for these asymmet-

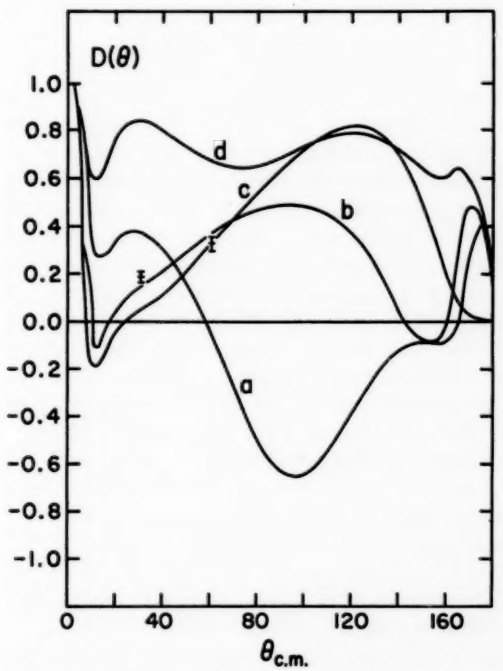

FIG. 1. Depolarization parameter $D(\theta)$ in p-p scattering at 210 Mev, with the predictions of phase shift solutions a through d.

ries, separate values of $D(\theta)$ for right and left scatterings were calculated from the observed asymmetries and the p-p polarization data[5] at this energy. The values of $D(\theta)$ obtained for left and right scatterings were then combined statistically, giving the following result:

$$D(30^\circ) = 0.19 \pm 0.02,$$

$$D(60^\circ) = 0.33 \pm 0.03.$$

The errors include the above-mentioned uncertainty in alignment. Figure 1 shows the experimental points along with the predictions[1] of solutions a through d. Our preliminary values of $D(\theta)$ at 210 Mev strongly substantiate the choice[1] of solution b or c and indicate some preference for b over c. With the $D(\theta)$ values at 310 Mev, the data yield an over-all energy dependence of $D(\theta)$ which favors the $D(\theta)$ measurement at 150 Mev by the Harvard group[6] rather than the Harwell result.[7]

The accuracy of the data is being improved and the work extended to other angles.

*Work done under the auspices of the U. S. Atomic Energy Commission.

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[2]P. Cziffra, M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. 114, 880 (1959).

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[4]L. Wolfenstein, Phys. Rev. 96, 1654 (1954).

[5]For these data, one should refer to reference 1.

[6]C. F. Hwang, T. R. Ophel, E. H. Thorndike, Richard Wilson, and N. F. Ramsey, Phys. Rev. Letters 2, 310 (1959). Also see C. F. Hwang, thesis, Harvard University, 1959 (unpublished).

[7]For the Harvard-Harwell discrepancy in $D(\theta)$ at 150 Mev, refer also to A. E. Taylor and E. Wood, 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958).

[8]E. Heer, J. Tinlot, W. Gibson, and A. England, Bull. Am. Phys. Soc. 3, 204 (1959).

CAPTURE OF K^- MESONS FROM HIGH S ORBITALS IN HELIUM*

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It would be quite interesting to know from which atomic orbital a K^- meson will be captured after stopping in liquid helium. For, if it can be established that, in all likelihood, the capture of the K^- meson occurred from S states, a study of the angular distribution of the mesonic, two-body decay of the hyperfragment ${}_\Lambda He^4$ or ${}_\Lambda H^4$ can determine the spin of the hyperfragment. If the spin of the ${}_\Lambda H^4$ or ${}_\Lambda He^4$ turns out to be zero, then the experiment of Block et al.[1] determines that the K^--Λ parity is odd.

The problem for helium is completely different and quite a bit more complicated than that for hydrogen, studied previously.[2,3] As a consequence, the result is not as conclusive as that for hydrogen, although it does indicate that S-state capture will again predominate.

We will outline qualitatively, here, the principal processes which occur after K^- mesons are stopped in a liquid helium bubble chamber.[4] When the K^- meson is first captured by a helium atom (into a state which best overlaps the wave function of the electron it replaces, i.e., with principal quantum number $n \sim 30$), it almost immediately de-excites and kicks out the remaining electron by the usual Auger process. The state to which it must fall about the unshielded alpha particle in order to release the 24.56 ev electron ionization energy has $n \sim 27$, and an average radius of $\sim 0.4 a_{el}$ (electronic Bohr radius). This is substantially smaller than the electron's average radius in a helium atom, and so, the $(K^-, \alpha)^+$ atom looks roughly like a proton to the surrounding helium atoms. In particular, it is energetically forbidden to pick up another electron.[5,6]

This, then, leads to an examination of the atomic and molecular processes in which a $(K^-, \alpha)^+$ atom can participate.[7] One possibility is that a metastable molecule might be formed with another helium atom. The $(p, \mathrm{He})^+$ molecule is well known,[8,9] although the three-body recombination problem involved in its formation has not been studied extensively in liquids.[10] However, while these recombination times are expected to be very short, it is not essential for our purposes, here, that the molecule actually form. Rather, we make only the simpler assumption that the $(K^-, \alpha)^+$ atom feels the molecular

potential (e.g. Fig. 1 of reference 8) while in collision with a nearby helium atom. Then, by using the Hellman-Feynman theorem[11] we can differentiate this curve to get the electric field at the $(K^-,\alpha)^+$ atom,[12] and we have recovered our Stark effect.

In order to have some idea of the time scale within which the various atomic processes must compete, we give first, the rate for direct capture from an (nS) state,

$$\Gamma_{\text{cap}}(nS) \simeq 2\times10^{19}/n^3 \text{ sec}^{-1}, \quad (1)$$

(where capture by neutrons is treated as equal to that by protons); next, the rate for direct capture of a K^- meson from an (nP) state via S-wave interaction with a nucleon (i.e., due to the finite size of the nucleus),

$$\Gamma_{\text{cap}}(nP) \sim 4\times10^{15}/n^3 \text{ sec}^{-1}. \quad (2)$$

Now, in order to estimate the average capture of a K^- meson from an $(n, m_l=0)$ level by the molecular Stark effect, we compute the rate for a fixed $(K^-,\alpha)^+$ - He distance, as if the molecular electric field were static. Then, we perform a suitable average over possible interatomic distances and impact parameters in an average collision between the $(K^-,\alpha)^+$ atom and another helium atom.[4] In this way, we define an average transition rate for this process, and find

$$\langle\Gamma_{\text{Stark}}(n, m_l=0)\rangle \sim 2\times10^5 n^6 \text{ sec}^{-1}. \quad (3)$$

A factor of $(1/n)$ has been incorporated, since there are n degenerate levels with the magnetic quantum number $m_l=0$ which are mixed and decay together in such a static external electric field, when all levels are populated statistically.[3] [A similar $1/n$ factor should multiply Eq. (2) to give the Stark capture from P states for $m_l=0$ before comparison is made with Eq. (3).]

In a similar way, we can find an average transition rate for the principally competing process, wherein the meson de-excites and directly ejects an electron from the nearby helium atom (the external Auger process).[13] By again assuming a statistical distribution, and considering only those transitions energetically possible and most favored by the selection rules, we get

$$\langle\Gamma_{\text{Aug}}(n=25)\rangle \sim 5\times10^{12} \text{ sec}^{-1},$$

$$\langle\Gamma_{\text{Aug}}(10)\rangle \sim 1\times10^{11} \text{ sec}^{-1},$$

$$\langle\Gamma_{\text{Aug}}(7)\rangle \sim 5\times10^{8} \text{ sec}^{-1}. \quad (4)$$

In general, the Stark capture rates from S states and from P states for $m_l=0$ would have to be modified by some factor which depended on the distribution of mesons in the levels of the atom, and on the directional properties of the electric field felt by the $(K^-,\alpha)^+$ atom.[3] However, two circumstances would seem to argue that the factor of $(1/n)$ incorporated in Eq. (4) is probably sufficient, if not already an overestimate of the reduction necessary because of the number of degenerate states involved in the Stark capture. The first is the fact that in liquid helium, positive ions lead to large clusters due to polarization forces.[14] This gives rise to weak electric fields whose directions change rapidly, which then destroys the $m_l=0$ selection rule for the ordinary Stark mixing. The second circumstance arises in the initial capture of the K^- meson by a helium atom. For, when this occurs, angular momentum barriers seem to prevent anything like a statistical distribution of mesons for the high l states, but rather, favor the population of low and intermediate angular momentum sublevels.[15] This kind of population will be accentuated by Auger transitions.[16] This has the effect not only of changing the factor $(1/n)$ in Eq. (4) to something like $(2/n)$, but also of decreasing the estimate of the external Auger rates, which were given in Eq. (4) for a statistical distribution.[17]

Hence, for the high n values considered here, $n \sim 20$-30, the S state capture via the molecular Stark effect will be ~10-50 times faster than the nearest competing process, the external Auger process. However, it should be pointed out that if the meson succeeds in getting to much lower n values, i.e., $n \sim 10$, where the external Auger process becomes negligible, the dominant reaction will be capture from P states via the Stark effect.

We would like to thank Professor L. S. Rodberg and Professor J. Sucher for many interesting discussions.

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[6]H. S. W. Massey and E. H. S. Burhop, Electronic and Ionic Impact Phenomena (Clarendon Press, Oxford, 1952).
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[10]See reference 5, Chap. X.
[11]H. Hellman, Einführung in die Quantenchemie (Franz Deuticke, Leipzig, 1937), p. 285; R. P. Feynman, Phys. Rev. 56, 340 (1939).
[12]L. Pauling, The Nature of the Chemical Bond (Cornell University Press, Ithaca, New York, 1960), 3rd ed.
[13]A closely related process wherein the meson shifts states, keeping the same principal quantum number, while the nearby electron is virtually excited, is found not to be important here and will be discussed in reference 4. This polarization-capture process was considered in another connection by M. A. Ruderman, Phys. Rev. 118, 1632 (1960). The transition rates for radiation are negligible for these highly excited states.
[14]G. Careri, U. Fasali, and F. S. Goeta, Nuovo cimento 15, 774 (1960).
[15]G. A. Baker, Jr., Phys. Rev. 117, 1130 (1960).
[16]M. Demeur, Nuclear Phys. 1, 516 (1956).
[17]The dipole matrix elements involved in the external Auger rates increase with increasing l for fixed initial and final principal quantum numbers. See H. A. Bethe and E. E. Salpeter, Quantum Mechanics of One- and Two-Electron Atoms (Springer-Verlag, Berlin, 1957), p. 264.

SYMMETRY BETWEEN MUON AND ELECTRON

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(Received July 13, 1960)

As is well known, muons and electrons appear to have identical couplings. Their masses are, however, different. Such a situation seems rather peculiar and has recently received much attention.[1] In this note we shall (1) define a formal operation of muon-electron symmetry; (2) show how the total Lagrangian, excluding weak couplings, can be written in a form exhibiting such a symmetry, if electromagnetic coupling is minimal; (3) show that it is impossible to satisfy such a symmetry when universal weak interactions are included, if only one neutrino exists; (4) show that it is possible to have such a symmetry in a two-neutrino theory; (5) point out the close connection of muon-electron symmetry to a principle forbidding the transformation of muons into electrons.

The present investigation is related to some recent papers[2-4] dealing with the elimination of particular muon-electron couplings. Of the above points, (2) is already contained in reference 3. We shall also make use of the general theorem of reference 4.

We first define a formal operation of muon-electron symmetry. We introduce a two-dimensional e-μ space, which we call L space (lepton space). The e-μ symmetry, or L symmetry, is performed by an unitary operator S, such that

$$S^{-1}\psi S = \sigma_1\psi, \tag{1}$$

where ψ is a vector in L space describing the electron and muon fields and σ_1 is a Pauli matrix in the usual notation. In the representation in which the components of ψ are e and μ the operation (1) just amounts to the substitution $e \rightleftarrows \mu$.

A general renormalizable Lagrangian, excluding weak interactions, can be written as[3,4]

$$\mathfrak{L} = -\bar{\psi}[\gamma\cdot\partial(A+\gamma_5 B)+(C+i\gamma_5 D)]\psi+\mathfrak{L}_\gamma+\mathfrak{L}_S, \tag{2}$$

where $\mathfrak{L}_\gamma$ is the free-photon Lagrangian, $\mathfrak{L}_S$ is the strong Lagrangian that we assume does not contain e or μ, and A, B, C, D are Hermitian matrices in L space.[5] The requirement of invariance under L symmetry implies that A, B, C, D all commute with σ_1.

A theorem, whose proof can be found in reference 4, states the existence of a nonsingular

matrix T in spin space and L space, such that by transforming according to

$$\psi = T\psi' \tag{3}$$

the Lagrangian takes its usual form in which the electron and muon components of ψ' are not coupled.[6] Thus there exist infinite choices of A, B, C, D that make the Lagrangian (2) manifestly L-symmetric. A particular choice is given in reference 3.

We now add weak interactions to the Lagrangian. We assume that e and μ are coupled identically in the $(1+\gamma_5)$ projection.[7] The matrix T is now restricted from the condition of giving a symmetric description also in terms of ψ'. Writing $T = aR + \bar{a}S$, where $a = \frac{1}{2}(1+\gamma_5)$ and $\bar{a} = \frac{1}{2}(1-\gamma_5)$ and R, S act in L space, such a condition restricts the form of R. R must be of the form

$$R = u + v\sigma_1 + w(\sigma_2 - i\sigma_3), \tag{4}$$

with u, v, w complex numbers.[8] One now sees directly that such a form of R is inconsistent with the assumption that A, B, C, D in Eq. (2) commute with σ_1. From (2) and (3) one sees that R and S must satisfy

$$R^\dagger(A+B)R = 1;\ S^\dagger(A-B)S = 1;\ S^\dagger(C+iD)R = M, \tag{5}$$

where $M = \frac{1}{2}(m_e + m_\mu) + \frac{1}{2}(m_e - m_\mu)\sigma_3$, to obtain, after transformation, the ordinary Lagrangian for muon and electrons. It follows from (5) that R has to satisfy the equations[9]

$$R^\dagger(A+B)R = 1;\ R^\dagger(A-B)^{-1}(C^2+D^2)R = M^2. \tag{6}$$

The first of Eqs. (6) implies $w = 0$ in (4).[10] But then it is impossible to satisfy the second of Eqs. (6) since the left-hand side commutes with σ_1 while M^2 does not.

A different situation occurs if one assumes the existence of two neutrinos, both left-handed: one, ν_e, coupled to the electron, and the other, ν_μ, coupled to the muon.[11] A simple transformation to obtain the desired symmetry consists in introducing new fields e', μ', ν_e', ν_μ', according to

$$e = \frac{1}{\sqrt{2}}(e'+\mu'), \quad \mu = \frac{1}{\sqrt{2}}(e'-\mu'),$$

$$\nu_e = \frac{1}{\sqrt{2}}(\nu_e'+\nu_\mu'), \quad \nu_\mu = \frac{1}{\sqrt{2}}(\nu_e'-\nu_\mu').$$

The total Lagrangian assumes the symmetric form

$$\mathfrak{L} = -\bar{e}'(\gamma\cdot\partial + m_+)e' - \bar{\mu}'(\gamma\cdot\partial + m_+)\mu' + m_-[(\bar{e}'\mu') + (\bar{\mu}'e')]$$
$$-\bar{\nu}_e'\gamma\cdot\partial\nu_e - \bar{\nu}_\mu'\gamma\cdot\partial\nu_\mu + G[(\bar{e}'\gamma_\lambda a\nu_e') + (\bar{\mu}'\gamma_\lambda a\nu_\mu')$$
$$+ \ldots][(\bar{\nu}_e'\bar{a}\gamma_\lambda e') + (\bar{\nu}_\mu'\bar{a}\gamma_\lambda\mu') + \ldots]$$
$$+ (\text{other terms not involving leptons}). \tag{7}$$

Here $m_\pm = \frac{1}{2}(m_e \pm m_\mu)$, G is the weak-coupling constant, and the contribution to the weak current from baryon and meson terms has not been written down explicitly. The Lagrangian (7) is written for the usual formulation of the A - V theory.[7] Of course, L symmetry here involves also an exchange of ν_e with ν_μ.

Finally we come to the last of the four points mentioned in the introduction. According to general principles we expect that a selection rule be connected to the possibility of L symmetry. One sees that $\mathcal{S}$ in (1) can be taken to satisfy $\mathcal{S}\mathcal{S}^\dagger = 1$, and $\mathcal{S}^2 = 1$, and therefore it is Hermitian, with eigenvalues ± 1. If L symmetry is satisfied, states with eigenvalue $+1$ cannot transform into states with eigenvalue -1. What is the physical meaning of this conservation law? From (1) and (3) one notices that $\mathcal{S}$ can also be represented by a matrix $T^{-1}\sigma_1 T$ acting on ψ'. Such a matrix has the following properties: (a) it is traceless, (b) its square is unity, and (c) it must commute with M, because of the invariance of $\mathfrak{L}$. Therefore it can only be $\pm\sigma_3$. It is now evident that the conservation law is one that forbids a muon to transform into an electron and vice versa (unless other particles such as ν_e and ν_μ, bearing quantum number $\mathcal{S}$, are also emitted or absorbed). We may call this law the "law of muonic number conservation". Such a law is not satisfied in the one-neutrino theory and this simple observation may actually be taken as an independent proof of our statement (3) that we derived above by direct algebraic verification. This remark also illustrates the role of minimal electromagnetic coupling in our statement (2), since, by nonminimal coupling, μ-e transitions could well occur.

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[1]See, e.g., J. Schwinger, Ann. Phys. 2, 407 (1957); M. Goldhaber, Phys. Rev. Letters 1, 467 (1958).

[2]N. Cabibbo and R. Gatto, Phys. Rev. 116, 1134 (1959).

[3]G. Feinberg, P. Kabir, and S. Weinberg, Phys. Rev. Letters 3, 527 (1959).

[4]N. Cabibbo, R. Gatto, and C. Zemach, Nuovo cimento 16, 168 (1960).

[5]The μ, e couplings with photons are here obtained,

according to minimal electromagnetic interaction, through the replacement $\partial/\partial x_\mu \rightarrow \partial_\mu = (\partial/\partial x_\mu - ieA_\mu)$. The matrix $A + \gamma_5 B$ is positive definite to assure a positive definite energy.

[6]The physical equivalence implied by (3) is easily seen by considering the mapping of the Hilbert space effected by a nonsingular matrix τ such that $\tau^{-1}\psi\tau = T\psi$. If P_μ' is the total energy-momentum vector constructed from ψ', the operator $P_\mu' P_\mu'$ has eigenvalues $-m_\mu^2$, which correspond to the electron and the muon. However, $P_\mu P_\mu$, where P_μ is constructed from ψ, must have the same eigenvalues since $P_\mu = \tau^{-1} P_\mu' \tau$.

[7]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958); E. C. G. Sudarshan and R. E. Marshak, Phys. Rev. 109, 1860 (1958).

[8]Arbitrary phase changes of the electron and muon fields are unessential to the conclusion.

[9]One notices that $A-B$ is positive definite because $A + \gamma_5 B$ is such, and that A, B, C, D all commute between themselves since they all commute with σ_1.

[10]Write $A + B = p + q\sigma_1$, and take $\chi_\pm = \begin{bmatrix} 1 \\ \pm 1 \end{bmatrix}$ as orthogonal base vectors. One finds $R^\dagger(A+B)R\chi_- = (p-q)(u-v)[(u^*-v^*)\chi_- + 2iw^*\chi_+] = \chi_-$, from which $w = 0$.

[11]A. Salam, Proceedings of the Seventh Annual Conference on High-Energy Nuclear Physics, 1957 (Interscience Publishers, New York, 1957), p. IX, 44; B. Pontecorvo, J. Exptl. Theoret. Phys. (U.S.S.R.) 37, 1751 (1951).

ERRATUM

EFFECTS OF PION-PION INTERACTION IN ELECTROMAGNETIC PROCESSES. L. M. Brown and F. Calogero [Phys. Rev. Letters 4, 315 (1960)].

Expressions (8) and (9) are too large by a factor 2, as are also Figs. 1 and 2 derived from them. This error was kindly indicated to the authors by Professor L. Michel. The same error appears also in older results on the spin-zero vacuum polarization[1] as has been noted previously.[2]

[1]R. P. Feynman, Phys. Rev. 76, 769 (1949).
[2]A. I. Akhiezer and V. B. Berestetsky, Quantum Electrodynamics (State Technico-Theoretical Literature Press, Moscow, 1959), 2nd ed.

ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

SOME RECENT EXPERIMENTAL TESTS OF THE "CLOCK PARADOX." C. W. Sherwin, Department of Physics and the Coordinated Sciences Laboratory, University of Illinois, Urbana, Illinois (Received March 28, 1960; revised manuscript received May 23, 1960).

Recent experiments by Pound and Rebka on the temperature dependence of the Mössbauer effect in Fe^{57}, and by Hay, Schiffer, Cranshaw, and Egelstaff using an Fe^{57} absorber on a rotating drum are shown to provide the first direct experimental verification of the time-keeping properties of accelerated clocks such as occur in the classic "clock paradox" of relativity. In the experiment by Pound and Rebka, the thermal vibrations of the lattice impart rms velocities of about $10^{-6}c$, and nearly continuous, randomly-oriented accelerations of the order of $10^{16}g$ to both the source and the absorber nuclei. In the experiment by Hay et al. the acceleration of the absorber was $6\times10^{4}g$. The photon provides continuous communication of time data between the two nuclei for the duration of the "journey" (the emission time of the quantum). In each case the observed fractional frequency shift $\Delta f/f_0$ which occurs between the source and the absorber is found to be $-v_s^2/2c^2+v_a^2/2c^2$, where v_s and v_a are the rms velocities of the source and the absorber nuclei, respectively. These results are in quantitative agreement with the generally accepted calculations for the "clock paradox," in which two clocks pursue independent paths (at least one of which involves accelerations) in a common inertial frame, but are compared at two or more points where they coincide in space and time. The temperature-dependent experiments also demonstrate that accelerations of the order of $10^{16}g$, arising from lattice vibrations produce no intrinsic frequency shift in Fe^{57} nuclei to an accuracy exceeding 1 part in 10^{13}.

EQUATION OF STATE AND PHASE TRANSITION OF THE SPHERICAL LATTICE GAS. Walter Pressman, U. S. Army Signal Research and Development Laboratories, Fort Monmouth, New Jersey, and Joseph B. Keller, Institute of Mathematical Sciences, New York University, New York, New York (Received April 6, 1960).

The spherical lattice gas is a modification of the ordinary lattice gas in which the occupation number of each cell is permitted to be any real number rather than ±1. However, the sum of squares of the occupation numbers is required to equal the number of cells. This permits one to evaluate the partition function by integrating over the surface of a certain sphere rather than by summing over lattice points on that surface. The partition function and the equation of state of the gas are evaluated in this way. It is found that in three dimensions the gas condenses, but not in one or two dimensions. Graphs of the phase transition curve and of the isotherms in three, two, and one dimension are presented.

The analytical work is simplified by taking advantage of the relationship between the properties of the lattice gas and of the Ising model of a ferromagnet. This relationship, demonstrated by Yang and Lee for the ordinary lattice gas and Ising model, also applies to the spherical lattice gas and the spherical model of a ferromagnet. The properties of the latter have been evaluated by Berlin and Kac. Graphs of the isotherms of the spherical model of the magnet, which were found in the course of the work, are also presented.

APPROXIMATE ANALYTIC APPROACH TO THE CLASSICAL SCATTERING PROBLEM. Guy W. Lehman and Kenneth A. Shapiro,* Atomics International, Canoga Park, California (Received May 25, 1960).

An approximate analytic approach to the problem of determining differential scattering cross sections for classical central-field repulsive forces is described. It is shown that the impact parameter, b, can be approximated by $b=R\cos(\theta/2)$, where R is approximately the distance of closest approach and θ is the scattering angle in the center-of-mass system. A simple approximation gives the potential energy of interaction between two atoms as $V(R)=2E\times\sin(\theta/2)$, where E is the energy in the center-

of-mass system. Simple analytic expressions for the differential scattering cross section, σ, are derived from the above two relationships for three special cases of a two-parameter screened Coulomb potential energy, $V(R)=Z_1Z_2e^2A\exp(-pAR)\times[1-\exp(-AR)]^{-1}$, where z_ie is the charge on the ith atom, A^{-1} is a screening radius, and p is an adjustable parameter which is restricted to 1/2, 1, and 2 in this paper.

A new and improved method for calculating σ exactly is also discussed and is used to compute the exact behavior of σ for $p=1$. A table is presented which allows one to compare the exact and approximate σ's for $p=1$ over a wide range of energy and scattering angles. The agreement is particularly good for large energy transfer.

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SPECIFIC HEAT OF THORIUM AT HIGH TEMPERATURES. Duane C. Wallace, Institute for Atomic Research and Department of Physics, Iowa State University of Science and Technology, Ames, Iowa (Received May 20, 1960).

The specific heat and electrical resistivity of high-purity thorium have been measured from room temperature to 1000°C. The specific heat was measured by an electrical pulse-heating method. The results have been analyzed in terms of additive lattice and electronic specific heats.

OPTICAL CONSTANTS OF SILICON IN THE REGION 1 TO 10 ev. H. R. Philipp and E. A. Taft, General Electric Research Laboratory, Schenectady, New York (Received May 16, 1960).

The reflectance, $|r(\lambda)|^2$, of single-crystal silicon was measured in the range 1 to 11.3 ev. The phase, $\theta(\lambda)$, was computed from these data using the Kramers-Kronig relation between the real and imaginary parts of the complex function $\ln r=\ln|r|+i\theta$. The optical constants, n and k, were then determined from the Fresnel reflectivity equation. The real part of the refractive index, n, shows a sharp maximum of magnitude 6.9 at 3.3 ev. The extinction coefficient, k, shows maxima of magnitude 3.1 at 3.5 ev and 5.1 at 4.3 ev; optical absorption above 3 ev is associated with the onset of strong direct transitions. The results indicate that much useful information, applicable to band structure calculations for both silicon and germanium, could be obtained from limited reflectance studies (2 to 5 ev) on Ge-Si alloys.

EFFECTS OF ILLUMINATION UPON SODIUM CHLORIDE THERMOLUMINESCENCE. A. E. Stoddard, California Research Corporation, La Habra, California (Received May 17, 1960).

Illumination of a sodium chloride crystal at liquid nitrogen temperature after x-ray irradiation at a higher temperature or higher temperature annealing has two effects upon its thermoluminescence: Glow peaks stable at the temperature of x-ray irradiation or annealing are diminished in intensity. Missing glow peaks, unstable at the temperature of x-ray irradiation or annealing, reappear in the glow curve. Both effects are greatest for illumination in the F band. The results are not consistent with existing models of the thermoluminescence process.

BROADENING OF THE MÖSSBAUER LINE. H. S. Snyder and G. C. Wick, Brookhaven National Laboratory, Upton, New York (Received May 27, 1960).

The thermal excitation of the solid which leads to a temperature-dependent shift of the Mössbauer line could conceivably cause a broadening of this line. We show here through a quantum-mechanical treatment, that for a perfect crystalline solid such a broadening does not occur.

CRITICAL FIELDS OF SUPERCONDUCTING TIN, INDIUM, AND TANTALUM. R. W. Shaw,* D. E. Mapother, and D. C. Hopkins, University of Illinois, Urbana, Illinois (Received May 23, 1960).

Precise ballistic-induction measurements of the critical field curves of tin, indium, and tantalum are reported. The measurements were made to provide more accurate data on the deviation of the critical field curves from the parabolic law. The results are generally within the range of uncertainty of earlier measurements. The main experimental error in the observed deviation now arises from uncertainty in the extrapolation of the measurements to 0°K from

the present lower limit of 1.1°K. Measurements at lower temperatures which will eliminate some of this uncertainty are to be desired.

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PHOTOEMISSION AND VALENCE BAND STRUCTURE OF ALKALI IODIDES. H. Philipp, E. Taft, and L. Apker, General Electric Research Laboratory, Schenectady, New York (Received May 16, 1960).

Energy distributions are described for external photoelectrons ejected from CsI by photons of energy 11.3 ev. All but about 5% of the photoelectrons emerge with kinetic energies between 1.5 ev and 5.3 ev. The conspicuous lack of electrons at lower energies is interpreted in terms of a valence band of width about 3 or 4 ev with an upper edge lying about 6 ev below the vacuum level. Related results are mentioned for RbI, KI, NaI, CsBr, and CsCl.

ANISOTROPIC SUPEREXCHANGE INTERACTION AND WEAK FERROMAGNETISM. Tôru Moriya,* Bell Telephone Laboratories, Murray Hill, New Jersey (Received May 25, 1960).

A theory of anisotropic superexchange interaction is developed by extending the Anderson theory of superexchange to include spin-orbit coupling. The antisymmetric spin coupling suggested by Dzialoshinski from purely symmetry grounds and the symmetric pseudo-dipolar interaction are derived. Their orders of magnitudes are estimated to be $(\Delta g/g)$ and $(\Delta g/g)^2$ times the isotropic superexchange energy, respectively. Higher order spin couplings are also discussed. As an example of antisymmetric spin coupling, the case of $CuCl_2 \cdot 2H_2O$ is illustrated. In $CuCl_2 \cdot 2H_2O$, a spin arrangement which is different from one accepted so far is proposed. This antisymmetric interaction is shown to be responsible for weak ferromagnetism in α-Fe_2O_3, $MnCO_3$, and CrF_3. The paramagnetic susceptibility perpendicular to the trigonal axis is expected to increase very sharply near the Néel temperature as the temperature is lowered, as was actually observed in CrF_3.

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SIMPLIFIED SELF-CONSISTENT FIELD EQUATIONS WITH CORRELATIONS. S. Olszewski, Electrode Processes Laboratory, Institute of Physical Chemistry, Polish Academy of Sciences, Warsaw, Poland (Received January 25, 1960; revised manuscript received May 27, 1960).

It is pointed out that the correlation hole has, qualitatively, features analogous to the exchange one; and to the correlation, there corresponds a potential of the conventional electrical character. We can further obtain—in the case of the free-electron gas—the correlation potential averaged over all the electron states, in close correspondence with the averaged exchange potential of Slater. It is further reasonable to use this potential as the averaged correlation potential of the arbitrary many-electron system where the density of the free-electron gas is replaced by the density of actual charge in the system.

Adding the averaged correlation potential as well as the averaged exchange correlation potential to the Hartree-Fock operators, we obtain the self-consistent field equations with correlations.

SCATTERING ANISOTROPIES IN n-TYPE SILICON. Donald Long and John Myers, Honeywell Research Center, Hopkins, Minnesota (Received May 23, 1960).

Measurements have been made of magnetoresistance effects in several relatively pure samples of n-type silicon for the purpose of obtaining information on scattering anisotropies. The results indicate that the ratios of relaxation times parallel and perpendicular to a constant-energy-spheroid axis in the six-valley conduction band of silicon are $(\tau_\parallel/\tau_\perp) \approx \frac{2}{5}$ for acoustic-mode intravalley lattice scattering and $(\tau_\parallel/\tau_\perp) > 1$ for ionized-impurity scattering. Intervalley lattice scattering, important at higher temperatures, is isotropic.

STEADY-STATE DISTRIBUTION FUNCTION IN DILUTE ELECTRON CASES. Daniel C. Mattis, Research Laboratory, International Business Machines Corporation, Poughkeepsie, New York (Received April 15, 1960).

It is usually assumed that optically created carriers in a photoconductor rapidly thermalize

to a Boltzmann distribution, regardless of the generation and recombination mechanisms. However, it can be shown that this distribution which is characteristic of thermodynamic equilibrium is incompatible with the requirements of steady state. A variational principle is introduced to find the steady-state distribution, which is found to approach the Maxwell-Boltzmann function in the limit of strong thermal scattering. Interband scattering is found to be potentially a strong thermalizing influence, in addition to the intraband scattering usually considered. For a simple model semiconductor, significant deviations from the Boltzmann distribution are found to be possible at temperatures below a few degrees Kelvin. This result is then discussed in connection with certain experiments on germanium.

CROSS RELAXATION IN DILUTE PARAMAGNETIC SYSTEMS. A. Kiel, Radiation Laboratory, The Johns Hopkins University, Baltimore, Maryland (Received May 16, 1960).

The theory of cross relaxation developed by Bloembergen, Shapiro, Pershan, and Artman is applied to the case of highly diluted paramagnetic salts. We find that the second moment for cross relaxation between two diluted paramagnetic species may be far greater than the sum of the second moments of individual lines which would be obtained from ordinary linewidth measurements. We can thus explain the dominance of cross relaxation in cases where the separation between the interacting resonances is up to twenty time the sum of the diluted linewidths.

SELF-DIFFUSION OF THE CHLORIDE ION IN SODIUM CHLORIDE. Neal Laurance,* Department of Physics, University of Illinois, Urbana, Illinois (Received May 18, 1960).

The diffusion coefficient of Cl^{36} in NaCl has been measured in the temperature range from 520° to 740°C. Diffusion was measured in pure Harshaw crystals and in crystals containing from 0.01 to 0.1 mole percent calcium. The diffusion coefficient in pure crystals is represented by the equation $D = 56 \exp(-2.12 \text{ ev}/kT)$ cm²/sec. The diffusion coefficient in crystals containing calcium was smaller than that measured in pure crystals by a factor of from 5 to 10, and had an activation energy of 2.5 ev. Over the range of impurity concentration employed, the diffusion coefficient was insensitive to differences in calcium concentration. The results are discussed in terms of motion of free negative-ion vacancies and of vacancy pairs. Possible complicating effects of dislocation lines on the diffusion coefficient are also considered.

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LATTICE ANHARMONICITY AND OPTICAL ABSORPTION IN POLAR CRYSTALS: I. THE LINEAR CHAIN. A. A. Maradudin* and R. F. Wallis, U. S. Naval Research Laboratory, Washington, D. C. (Received November 30, 1959; revised manuscript received April 21, 1960).

A calculation of the optical absorption spectrum of an anharmonic one-dimensional lattice of alternating positively and negatively charged particles of different masses is carried out using two different approaches: the recent theory of Born and Huang, and ordinary second order time-dependent perturbation theory. Closed form expressions for the absorption spectrum are obtained for both low and high temperatures. It is found that subsidiary absorption peaks may be expected at frequencies other than the dispersion frequency ω_d. At high temperatures the absorption at ω_d varies as T^{-2} or T^{-3} depending on how certain thermal averages are carried out. Predictions based on these results are made regarding which features of the present spectra can be expected to persist for three-dimensional lattices.

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DISPERSION RELATIONS FOR BLOCH FUNCTIONS. Peter E. Kaus and W. K. R. Watson, Physics Department, University of Southern California, Los Angeles, California (Received April 29, 1960; revised manuscript received June 16, 1960).

It is shown that the Floquet factor $e^{ik(E)a}$ is analytic in the upper half complex energy plane, thus enabling a set of four dispersion relations to be derived from this expression as a direct result of the application of Cauchy's theorem.

These relations are characterized by their ability to relate the wave number k at one energy to the wave number at all others. In particular, the imaginary part of the wave number, k_i, in the forbidden gap may be equated to an integral of a function of the real part of the wave number, k_r, over allowed energies. As an application of these dispersion relations, a theorem regarding the location of the branch points has been established.

BAND STRUCTURE OF TRANSITION METALS AND THEIR ALLOYS. John B. Goodenough, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts (Received September 18, 1958; revised manuscript received May 20, 1960).

A departure from the conventional approach to the energy-band problem is achieved in three ways. First, it is noted that there is a critical atomic separation $R_c \leq (2.9 \pm 0.1)$ A such that for $R < R_c$ electrons from atomic $3d$ orbitals that are directed along a ligand must be treated as collective electrons, while for $R > R_c$ the corresponding electrons are localized, Heitler-London electrons. Since the $3d$ wave functions are anisotropic, this implies that there may be localized and collective $3d$ electrons simultaneously present. Second, it is pointed out that localized electrons obey Hund's rule and may therefore contribute an atomic moment. This means the corresponding energy levels, or narrow bands, are split into discrete sub-bands. Any moment from collective $3d$ electrons is induced by the simultaneously present localized electrons via intra-atomic exchange. Third, it is asserted that if nearest-neighbor antiferromagnetic order can be propagated throughout a lattice and the nearest-neighbor-directed $3d$ orbitals are half or less filled, the collective electrons ($R < R_c$) can be stabilized by bonding-band formation. If the orbitals are more than half filled, the "extra" electrons cannot be stabilized by antiferromagnetic correlations between nearest neighbors. If antiferromagnetic, nearest-neighbor order is not possible, the electrons form a conventional metallic band. These observations provide sharp criteria for Pauli paramagnetism, antiferromagnetism, ferrimagnetism, and ferromagnetism in transition metals and their alloys. They are used to introduce electron correlations explicitly into the construction of qualitative energy diagrams from which semiempirical density-of-states curves are constructed. The resulting model is shown to provide a consistent interpretation of phase stability, magnetic properties, electronic specific heats, Hall effect data, and form-factor measurements for the bcc and close-packed transition metals of the first long period and their alloys. The model is only partially successful for elements of the second and third long periods.

FERRIMAGNETIC RESONANCE IN RARE-EARTH-DOPED YTTRIUM IRON GARNET: FIELD FOR RESONANCE. J. F. Dillon, Jr., and J. W. Nielsen, Bell Telephone Laboratories, Murray Hill, New Jersey (Received May 18, 1960).

Resonance experiments have been performed on yttrium iron garnet crystals doped with each of the rare earth ions except Lu, Gd, and Pm. Except for Ce these are thought to replace Y as trivalent ions. This paper presents measurements of the field for resonance in the (110) plane at 1.5°K for each of these samples. In several cases there are also data up to about 25°K. Except for Ce, Eu, and Tm the curve H_{res} in (110) at 1.5°K shows characteristic structure. For Pr, Ho, and Tb this structure is dominated by very narrow peaks in H_{res}. For Nd, Sm, Dy, and Er it is relatively broad in angle. For Yb most of the structure is relatively broad, but there is a very sharp but small spike in H_{res}. Only in the case of Tb and Yb do sharp spikes appear at other than symmetry directions. In some cases the height of the peaks falls off rapidly with increasing temperature starting at the lowest temperatures, but in other cases it does not change at first, then falls off.

THEORY OF VACANCY ANNEALING IN IMPURE METALS. A. C. Damask* and G. J. Dienes, Brookhaven National Laboratory, Upton, New York (Received May 18, 1960).

The general equations for the annealing of vacancies in metals containing impurities to which the vacancies can be attached have been solved on an analog computer for a wide variety of parameters. The computer results show that some simplifying assumptions may be made

which permit the general equations to be solved analytically. It is shown that for many physically interesting cases of vacancy migration the decay curve is exponential, and the decay constant is related to, but not equal to, the rate constant for vacancy migration. It is further shown that only experiments performed on zone-refined metals can give the correct vacancy migration energy, and that impurity contents as low as 10^{-5} can seriously affect the results. Experimental methods and calculations are discussed which can be used to measure the binding energy of vacancies to impurities in metals prepared by controlled doping.

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MAGNETIC SUSCEPTIBILITY OF p-TYPE Ge. R. Bowers* and Y. Yafet,† Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received May 26, 1960).

The magnetic susceptibility of p-type Ge has been measured for a range of extrinsic carrier densities extending from 5×10^{17} cm^{-3} to 5×10^{20} cm^{-3}. Measurements were made in the temperature range 300°K to 1.3°K. The degenerate hole susceptibility was determined from the data. At the lower carrier densities, the data depart appreciably from the Landau-Peierls value; above 10^{20} cm^{-3} the data exhibit features due to the population of the split-off band. From the experimental results, it is estimated that the Fermi level touches the minimum of the split-off band at a carrier density of 1.3×10^{20} cm^{-3}. A qualitative discussion is given of the factors determining the susceptibility, including band degeneracy and spin-orbit coupling; a detailed quantitative analysis is not attempted.

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VARIATIONAL TREATMENT OF WARM ELECTRONS IN NONPOLAR CRYSTALS. I. Adawi,* RCA Laboratories, Princeton, New Jersey (Received May 16, 1960).

Deviations from Ohm's law in nonpolar crystals are treated for weak fields by the variational method. A simple band structure is assumed. Scattering by both acoustical and optical phonons, and ionized impurities is included. It is shown that the influence of optical phonons on the field-dependent mobility (E^2 term where E is the electrostatic field) is maximum for a temperature which corresponds approximately to the optical phonon energy. The field-dependent mobility is highly sensitive to ionized impurity scattering as in the case of acoustical phonons alone. Finally, the convergence of the variational method is established in limiting cases using as a representation a set of polynomials which are orthonormal with respect to the collision operator. Extensive calculations are given for electrons in germanium and comparison with experiment is discussed.

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TRANSPORT IN METALS. II: THE EFFECT OF THE PHONON SPECTRUM AND UMKLAPP PROCESSES AT HIGH AND LOW TEMPERATURES. M. Bailyn, Department of Physics, Northwestern University, Evanston, Illinois (Received April 22, 1960).

A calculation of the nonmagnetic transport coefficients of the alkali metals is made, with improvements designed to take into account the effect of the phonon spectrum on both the normal and umklapp regions of scattering. The phonon equations of motion are solved numerically to obtain a spectrum sample, and spectrum averages are then computed in a manner similar to specific heat calculations, although we do not need or compute the density of states directly. No average Debye temperatures are used, but rather the sums are obtained in terms of certain combinations of the elastic constants, which in principle are measurable. Also, improvements on the shielding part and on the ion part of the electronic matrix element are calculated and discussed. The results show that umklapp processes are important down to the lowest measurable temperatures in the ideal component of the electrical and thermal resistivities, being completely dominant in the former. The low-temperature temperature dependence is therefore determined mainly from the umklapp term, which can show a faster variation than T^5 in the electrical resistivity, as is actually observed in sodium. The transverse phonon vibrations dominate the contributions at all temperatures

and even the nonumklapp term at low temperatures. The computations give absolute magnitudes for the resistivities which are much too large at low temperatures. This is tentatively attributed in part at least to a spectrum which perhaps exaggerates the anisotropy of the transverse phonons. General expressions for the transport coefficients are calculated via the Kohler variational principle which are not restricted to the model of spherical energy surfaces.

LOWER BOUNDS FOR EIGENVALUES WITH APPLICATION TO THE HELIUM ATOM. Norman W. Bazley, Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland (Received May 12, 1960; revised manuscript received June 27, 1960).

A method is derived for finding lower bounds to the energy levels of the Schrödinger equation. This method is applied to the helium atom. The best lower bounds thus obtained are -3.063_7 and -2.165_5 atomic units for the energies $E(1\,^1S)$ and $E(2\,^1S)$, respectively. If our lower bound for $E(2\,^1S)$ is used together with the best published values of $\langle H\psi, \psi\rangle$ and $\langle H\psi, \psi\rangle$ of the ground state, a rigorous lower bound -2.9037474 atomic units is found for $E(1\,^2S)$.

COLLECTION OF IONS PRODUCED BY ALPHA PARTICLES IN AIR. Z. Bay and H. H. Seliger, The National Bureau of Standards, Washington, D. C. (Received May 16, 1960).

In the measurement of the ionization caused by alpha particles in air, recombination effects between slowly moving positive and negative ions (the latter formed by electron attachment to oxygen) have to be considered. The usual procedure in such measurements is to determine the saturation current (by extrapolation of reciprocal current versus reciprocal voltage curves to infinite field strength) according to the Jaffé theory. A paper by Wingate, Gross, and Failla has cast doubt on the validity of this extrapolation technique, in that the authors propose a field-independent part of the recombination amounting to 3.3% at atmospheric pressure in air. This proposal implied that all previous measurements of W_α for air were in error by this amount and that this error is a possible cause for the reported difference (3-4%) between W values for alpha and beta particles in air. In view of our own W measurements we felt compelled to re-examine this supposedly field-independent part of the recombination. Approximating the experimental conditions of Wingate, Gross, and Failla, we have not been able to reproduce their effect and our experiments demonstrate the validity of the usual extrapolation techniques.

HELIUM WAVE FUNCTION IN MOMENTUM SPACE. M. G. Henderson and Charles W. Scherr, Department of Physics, The University of Texas, Austin, Texas (Received May 9, 1960).

Approximate solutions to the integral Schrödinger equation in momentum space are obtained. The iteration scheme of Svartholm is used to obtain the first iterated wave function and the half-iterated energy. A wave function of the type

$$\phi = \sum C_{ij}[\exp(-\alpha_i p_1^{\,2} - \alpha_j p_2^{\,2}) + \exp(-\alpha_j p_1^{\,2} - \alpha_i p_2^{\,2})]$$

is employed to start the iteration procedure. The best energy value computed using a wave function with three nonlinear parameters is -2.8915 atomic units. This energy is to be compared with the result of a conventional variational calculation using the same wave function in coordinate space, -2.85112 atomic units.

UPPER BOUND ON TOTAL ELECTRON SCATTERING CROSS SECTIONS IN HYDROGEN. Nicholas A. Krall and E. Gerjuoy, John Jay Hopkins Laboratory for Pure and Applied Science, General Atomic, San Diego, California (Received May 16, 1960).

Dispersion relations for electron-hydrogen scattering are combined with existing scattering length calculations. The sign of the scattering length is shown to give an upper bound on $\int \sigma(k)dk$, where $\sigma(k)$ is the total cross section for scattering of electrons of incoming momentum $\hbar k$. Recent calculations of the scattering length are used to determine this limit. An experiment by Fite et al. satisfies this bound, agreeing with the recent calculations of the scattering length.

RESONANT SCATTERING OF X RAYS FROM MAGNESIUM AND SILICON. Ralph A. Tobin, Nucleonics Division, U. S. Naval Research Laboratory, Washington, D. C. (Received May 26, 1960).

Energy levels at 10.15 ± 0.06 Mev in magnesium and 11.40 ± 0.06 Mev in silicon have been studied using resonant scattering from a continuous spectrum of x rays by samples of these materials. Using tentative spin assignments and assuming no branching, level widths of 4.80(+1.6, -1.4) electron volts and 2.89(+1.0, -0.8) electron volts for magnesium and silicon, respectively, have been determined from absorption measurements.

ANGULAR DISTRIBUTION OF FRAGMENTS IN FISSION INDUCED BY Mev NEUTRONS. J. E. Simmons and R. L. Henkel, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received April 1, 1960).

A multiangle gas-filled counter has been used to measure the fragment angular distribution in fission induced by neutrons in the energy range $0.5 \leq E_N \leq 9$ Mev. The target nuclei used were: Th^{230}, U^{233}, U^{234}, U^{235}, U^{236}, U^{238}, Np^{237}, and Pu^{239}. In the cases of U^{233} and U^{235} the neutron energy range was extended to include energies between 14.8 and 23 Mev. The general features of these data are the following: The anisotropy—$(0°/90°)$ intensity ratio—has values between 1.1 and 1.2 depending on the target and is roughly independent of energy for E_N between 2 and 5.5 Mev. At higher energies a rise is observed such that at 7 Mev even-odd targets give values of anisotropy in the range 1.2 to 1.3 while even-even targets show greater values in the range 1.6 to 2.2. The anisotropy decreases somewhat by 9 Mev. Near thresholds for the even-even target nuclides considerable fluctuations of anisotropy are observed. The example of U^{236} at 0.85 Mev shows a new case of minimum intensity at 0°, the anisotropy being 0.64. In the energy region 2-4 Mev, the anisotropy of Pu^{239}, U^{233}, and U^{235} increases by a few percent from one to the next as the spin increases. This is contrary to simple theoretical expectations. These data have been compared to recent theoretical developments of the Bohr model as given by Griffin and by Halpern and Strutinski. The theory provides a satisfactory account of many features of the data.

DECAY OF Pm^{149} (53 hr). L. C. Schmid and S. B. Burson, Argonne National Laboratory, Argonne, Illinois (Received May 25, 1960).

The radioactive sources were obtained from neodymium oxide, enriched in Nd^{148}, irradiated in the Argonne reactor *CP*-5. After the 1.8-hr activity of Nd^{149} had decayed away, the 53-hr activity in $_{61}Pm^{149}$ remained. Some of the samples were purified by ion-exchange-column techniques before being studied. The radiations were investigated with a 180°-focusing beta-ray spectrometer and the Argonne 256-channel scintillation coincidence spectrometer. The magnetic spectrometer resolved two components of the beta spectrum at 1.064 ± 0.008 and 0.784 ± 0.010 Mev. The beta rays were also studied by coincidence absorption techniques. In addition to the branch with a maximum energy of 0.784 ± 0.010 Mev, this method revealed two more at 0.47 ± 0.04 and 0.19 ± 0.04 Mev. The scintillation pulse-height spectrum of the gamma rays revealed the presence of three transitions at 0.850 ± 0.008, 0.582 ± 0.006, and 0.285 ± 0.001 Mev. A fourth gamma ray with an energy of 0.548 ± 0.006 Mev was found in coincidence measurements. These radiations are fitted into a decay scheme comprising the ground state and four excited states in Sm^{149} at 0.285, 0.582, 0.833, and 0.850 Mev. The log*ft* values, transition intensities, and possible spin and parity assignments are discussed.

GAMMA RAYS FROM THE PROTON BOMBARDMENT OF NATURAL SILICON. L. W. Seagondollar, Gale I. Harris, and L. Kasturi Rangan, Department of Physics and Astronomy, University of Kansas, Lawrence, Kansas (Received February 26, 1960; revised manuscript received June 2, 1960).

The gamma-ray yield curve was observed when thin targets of natural silicon were bombarded with monoergic protons in the energy range of 300 to 1840 kev. In order to take small steps in proton energy, a target potential modulation technique was used. Fifty-five resonances were observed, all but fifteen of which have been observed elsewhere using targets enriched in Si^{29} or Si^{30}. The fifteen resonances at 369, 1096, 1134, 1204, 1290, 1382, 1472, 1484, 1507, 1570, 1598, 1617, 1625, 1630, and 1653 kev are presumed due to the $Si^{28} + p$ reaction.

TRANSITION INTENSITIES IN THE Tl^{208} BETA DECAY, THE $Bi^{212} - Po^{212}$ DECAY SCHEME, AND THE Bi^{212} BRANCHING RATIO. G. Schupp, H. Daniel,* G. W. Eakins, and E. N. Jensen, Institute for Atomic Research and Department of Physics, Iowa State University, Ames, Iowa (Received April 14, 1960).

Studies were made on the Pb^{212} (ThB) active deposit by means of gamma singles and beta-gamma, gamma-gamma, gamma-alpha, and gamma-gamma-alpha coincidence measurements. The singles and coincidence gamma-ray spectra were recorded on an RCL 256-channel analyzer, and an intermediate-image beta-ray spectrometer was used in the beta-gamma work. Beta intensities of 4.6 ± 0.2, 23.9 ± 0.8, 22.7 ± 0.7, 48.8 ± 2.7, and $< 0.5\%$ were obtained for the 1.04-, 1.29-, 1.52-, 1.80-, and 2.38-Mev groups, respectively, of the $Tl^{208} - Pb^{208}$ decay. Existence of the 1.800-Mev gamma ray in Po^{212} was established and $11.2 \pm 0.7\%$ of the $Bi^{212} - Po^{212}$ disintegrations were determined to go by way of the 0.727-Mev transition. Relative intensities of 11.1 ± 0.7, 1.7 ± 0.3, 0.66 ± 0.07, 0.16 ± 0.04, 0.99 ± 0.08, 0.49 ± 0.05, 2.8 ± 0.2, and 0.17 ± 0.03 were found for the 0.727-, 0.786-, 0.893-, 0.953-, 1.073-, 1.513-, 1.620-, and 1.800-Mev gamma rays, respectively, in Po^{212}. The ratio of alpha to total disintegrations for the Bi^{212} decay was measured to be 0.3596 ± 0.0006.

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LIFETIMES OF THE FIRST EXCITED STATES OF F^{17} AND O^{17}. J. V. Kane, R. E. Pixley, R. B. Schwartz, and A. Schwarzschild, Brookhaven National Laboratory, Upton, New York (Received May 20, 1960).

Using the gamma-gamma coincidence method and an electronic time-to-amplitude converter, the lifetimes of the first excited states in the mirror pair O^{17}(0.87 Mev) and F^{17}(0.50 Mev) have been measured. The reactions initiating the gamma cascades were $O^{16}(d, p)O^{17*}$ and direct proton capture in O^{16}. The time resolution of the coincidence circuit was good enough to allow the lifetime measurements to be made by direct observation of the exponential decay, rather than by measurement of the centroid shift. The measured mean life of O^{17*} is $(2.55 \pm 0.13) \times 10^{-10}$ sec; the measured mean life of F^{17*} is $(4.45 \pm 0.22) \times 10^{-10}$ sec. Both of these results are in reasonable agreement with earlier, less accurate values.

COLLECTIVE QUADRUPOLE EFFECTS IN LIGHT NUCLEI. B. James Raz, State University College on Long Island, Oyster Bay, New York (Received May 20, 1960).

The recent measurements of the lifetimes of the first excited states of the mirror nuclei O^{17} and F^{17} have raised new interest in the various theoretical interpretations of these lifetimes. In this work the weak-coupling collective model of Bohr and Mottelson is applied to these $E2$ transitions and to the similar $E2$ transitions that have been measured in N^{16}, F^{19}, and Ne^{19}. If harmonic oscillator radial wave functions are used in evaluating the radial integrals in the theory, the predictions match the experimental results for the $E2$ transition probabilities in N^{16}, O^{17}, F^{17}, and F^{19} and the quadrupole moment of O^{17}. The theoretical prediction is an order of magnitude smaller than the experimental result for transition probability of the first excited state of Ne^{19}.

SHELL MODEL AND Pb^{208}. J. C. Carter,* William T. Pinkston,† and William W. True, Palmer Physical Laboratory, Princeton University, Princeton, New Jersey (Received May 23, 1960).

The lowest odd-parity excited energy levels of Pb^{208} have been calculated by a shell-model approach considering a single proton or a single neutron to be excited out of the Pb^{208} core. Both a singlet-even plus triplet-even force and a Rosenfeld force were used as the two-particle interaction. A zero-range force was also considered. There were no other arbitrary parameters. The results with the various forces indicate that it is impossible to get a 3- state low enough to be interpreted as the observed 2.615-Mev 3- level. The results, therefore, support the conclusion that the 3- level at 2.615 Mev in Pb^{208} is primarily the result of a collective octupole oscillation.

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ENERGY LEVELS OF He^5 AND Li^5. L. D. Pearlstein, Y. C. Tang, and K. Wildermuth, Florida State University, Tallahassee, Florida (Received May 23, 1960).

The energies of the first three levels in He^5 are determined using a variational procedure. The various wave functions adopted incorporate alpha, triton, and deuteron correlations. It is determined that the ground state ($\frac{3}{2}$-) must be an alpha-neutron configuration whereas the ($\frac{1}{2}$-) level, which must also be described by this configuration, is not a true resonant state. The ($\frac{3}{2}$+) level at 16.69 Mev is shown to be a deuteron-triton configuration. The resultant energies and structures of these levels are in accord with the experimental situation. It should be stressed that our wave functions differ appreciably from the standard shell-model ones in the intermediate-coupling picture.

ATOMIC MASSES IN THE HEAVY-MASS REGION. V. B. Bhanot,* W. H. Johnson, Jr., and A. O. Nier, School of Physics, University of Minnesota, Minneapolis, Minnesota (Received May 19, 1960).

A six-inch double-focusing mass spectrometer has been employed to determine 61 mass doublets in the region of gadolinium to gold. The present results and other Minnesota mass data have been combined with nuclear reaction, β-decay, and α-decay energies in order to construct a mass table for more than 200 stable and radioactive isotopes in the region from samarium to radon. Total atomic binding energies as well as nucleon separation and pairing energies have been computed, wherever possible.

The present data confirm with greater detail the previously reported anomalies in the nucleon separation and pairing energies in the regions around 90 neutrons and 116 neutrons. The proton pairing energies are found to show rather pronounced "maxima" around $N=88$ and $N=116$, a behavior similar to the previously reported behavior of neutron pairing energies. The nature of the discontinuities in these two regions does not appear to follow the patterns found at major shell closures but seems to be caused by a change in the nuclear structure in these regions. It is known that such a change is indicated also by other nuclear properties.

Major discontinuities connected with the shell closures at $Z=82$ and $N=126$ are brought out in greater detail than has previously been possible. The mass data have also been employed for the study of isotopic assignments for several nuclear reactions in this region.

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$B^{10}(d,p)B^{11}$ REACTION AND THE CONFIGURATIONS OF B^{11}. O. M. Bilaniuk* and J. C. Hensel,† Harrison M. Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan (Received April 21, 1960).

A study of the states of B^{11} has been made by high-resolution measurements of angular distributions and relative intensities of proton groups from $B^{10}(d,p)B^{11}$. The use of Butler analysis and calculated reduced widths leads to the following spin and parity assignments for excited states: 2.14 Mev, 1/2-; 4.46 Mev, 5/2-; 5.03 Mev, 3/2-; 6.76 Mev, 7/2-; 8.57 Mev, (1/2+); 8.92 Mev, 5/2+; 9.19 Mev, 7/2+; 9.28 Mev, 5/2+. The states of B^{11} exhibit a subdivision into three classes. The first five levels from ground state to 6.76 Mev, all of odd parity, correspond closely to the theoretical predictions of Kurath. The next four states of 6.81, 7.30, 7.99, and 8.57 Mev presumably result from highly mixed configurations which is evidenced by their weak stripping intensities. The third group consists of three single-particle states of even parity at 8.92, 9.19, and 9.28 Mev. The latter two states form a jj-double level arising from the configuration $p^0 2s$.

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ATOMIC g_J VALUES FOR NEON AND ARGON IN THEIR METASTABLE $^3P_2$ STATES; EVIDENCE FOR ZERO SPIN OF $_{10}Ne^{20}$. A. Lurio, Yale University, New Haven, Connecticut, and International Business Machines Watson Laboratory, New York, New York, G. Weinreich,* Columbia University, New York, New York, and C. W. Drake, V. W. Hughes, and J. A. White,† Yale University, New Haven, Connecticut (Received May 24, 1960).

The gyromagnetic ratios of neon and argon in their metastable $^3P_2$ states have been measured

by the atomic beam magnetic resonance method. The results are $g_J(\mathrm{Ne}, {}^3P_2) = 1.500888 \pm 0.000005$ and $g_J(\mathrm{A}, {}^3P_2) = 1.500964 \pm 0.000008$, in agreement with the less precise optical spectroscopic measurements. Theoretical values, including radiative and relativistic effects, are $g_J(\mathrm{Ne}, {}^3P_2) = 1.50088$ and $g_J(\mathrm{A}, {}^3P_2) = 1.50095$, in good agreement with the experimental values. In addition, the Zeeman transition frequency for neon has been measured as a function of magnetic field to obtain evidence that the magnetic moment of Ne^{20} is less than 4×10^{-4} nuclear magneton and hence that the spin of Ne^{20} is probably zero.

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APPLICATION OF THE PHASE-SPACE QUASI-PROBABILITY DISTRIBUTION TO THE NUCLEAR SHELL MODEL. George A. Baker, Jr., University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico, Ian E. McCarthy,* School of Physics, University of Minnesota, Minneapolis, Minnesota, and Department of Physics, University of California, Los Angeles, California, and Charles E. Porter, School of Physics, University of Minnesota, Minneapolis, Minnesota (Received April 11, 1960).

The quantum mechanical joint position-momentum quasi-distribution function is applied to the nuclear shell model. By introducing approximate quasi-position and quasi-momentum variables, the quasi-distribution function is converted into a non-negative (and hence nonquasi) distribution. Numerical results are presented for one-dimensional and three-dimensional potentials leading in three dimensions to a non-isotropic nonindependent distribution with a predominance of low momenta at the nuclear surface. These results are in contrast with the usual Thomas-Fermi model and in addition provide a simple base for the discussion of direct nuclear reactions involving an average over many states of a residual nucleus for which linear momentum as opposed to angular momentum is a relevant quantity.

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HIGH-ENERGY ELECTRON-ELECTRON SCATTERING. Yung Su Tsai, Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California (Received May 10, 1960).

The radiative corrections to the electron-electron scattering to order α^3 are calculated for (1) the colliding beam experiment and (2) the experiment in which the target electron is at rest initially. The contributions from high-energy real photons are included. The two-photon exchange diagrams are found to give only negligible contributions to the cross sections after infrared cancellation. The effect due to the possible breakdown of quantum electrodynamics is discussed. A preliminary study on the electron-positron colliding-beam experiment involving various interactions is made. The vacuum polarizations involving heavier particles than an electron pair in the closed loop are investigated.

INFLUENCE OF BOSE-EINSTEIN STATISTICS ON THE ANTIPROTON-PROTON ANNIHILATION PROCESS. Gerson Goldhaber, Sulamith Goldhaber, Wonyong Lee, and Abraham Pais,* Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California (Received May 16, 1960).

Recent observations of angular distributions of π mesons in $\bar{p}$-p annihilation indicate a deviation from the predictions of the usual Fermi statistical model. In order to shed light on these phenomena, a modification of the statistical model is studied. We retain the assumption that the transition rate into a given final state is proportional to the probability of finding N free π mesons in the reaction volume, but express this probability in terms of wave functions symmetrized with respect to particles of like charge. The justification of this assumption is discussed. The model reproduces the experimental results qualitatively, provided the radius of the interaction volume is between one-half and three-fourths of the pion Compton wavelength; the dependence of angular correlation effects on the value of the radius is rather sensitive. Quantitatively, there seems to remain some discrepancy, but we cannot say whether this is due to experimental uncertainties or to some other dynamic effects. In the absence of information on π-π

interactions and of a fully satisfactory explanation of the mean pion multiplicity for annihilation, we wish to emphasize the preliminary nature of our results. We consider them, however, as an indication that the symmetrization effects discussed here may well play a major role in the analysis of angular distributions. It is pointed out that in this respect the energy dependence of the angular correlations may provide valuable clues for the validity of our model.

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ENERGY DETERMINATION OF HEAVY PRIMARIES IN NUCLEAR EMULSION. P. L. Jain,* Department of Physics, University of Chicago, Chicago, Illinois (Received May 20, 1960).

The "knock-on electron" method has been used to determine the energy per nucleon of the heavy primary particle. In this method, the energy of the primary particle is determined by measuring the emission angle and the energy of the knock-on electrons. The conditions for the reliable estimate of the primary energy by this method are discussed. This method is applied to 34 flat events, of primary energy between 3 and 30 Bev/nucleon and of charge $Z>4$, which make nuclear interactions and break up into two or more α particles. The energy of the primary particle obtained by knock-on electron method is then compared with the energy obtained by (i) opening angle of α particles and by (ii) relative scattering measurements of α particles. The results obtained by knock-on electron method are quite consistent, within the experimental error, with the results obtained by other methods.

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DECAY ASYMMETRY OF Σ^+ AND Λ^0 HYPERONS. Bruce Cork, Leroy Kerth, and W. A. Wenzel, Lawrence Radiation Laboratory, University of California, Berkeley, California, and James W. Cronin, Palmer Physical Laboratory, Princeton University, Princeton, New Jersey, and R. L. Cool, Brookhaven National Laboratory, Upton, New York (Received May 26, 1960).

Counter techniques have been used to measure decay asymmetries for polarized Λ^0 and Σ^+ hyperons. The Σ^+ hyperons were produced by 1.13-Bev/c π^+ in the reaction $\pi^+ + p \rightarrow \Sigma^+ + K^+$. The Λ^0 hyperons were produced by 1.00-Bev/c π^+ in the reaction $\pi^+ + d \rightarrow K^+ + \Lambda^0 + p$. The asymmetry for each hyperon decay mode was measured with a counter arrangement which detected separately charged pions and γ rays from neutral pions. The protons from the hyperon decay were also detected to confirm the identification of the decay modes. The product $\alpha\bar{P}$ was measured, where α is the asymmetry parameter and $\bar{P}$ is the average polarization of the hyperon. The following results were obtained for the various decay modes:

$$\Sigma^+ \rightarrow \pi^+ + n, \quad \alpha\bar{P} = +0.03 \pm 0.08;$$

$$\Sigma^+ \rightarrow \pi^0 + p, \quad \alpha\bar{P} = +0.75 \pm 0.17;$$

$$\Lambda^0 \rightarrow \pi^- + p, \quad \alpha\bar{P} = +0.55 \pm 0.06;$$

$$\Lambda^0 \rightarrow \pi^0 + n, \quad \alpha\bar{P} = +0.60 \pm 0.13.$$

The results for Σ^+ are fitted to the triangular relationship demanded by the $|\Delta I| = \frac{1}{2}$ rule, and absolute values for the asymmetries of $\Sigma^\pm$ decay are predicted. The ratio $\alpha(\Lambda^0 \rightarrow \pi^0 + n)/\alpha(\Lambda^0 \rightarrow \pi^- + p) = +1.10 \pm 0.27$ is in agreement with the value +1.00 predicted by the $|\Delta I| = \frac{1}{2}$ rule for hyperon decay.

POSSIBLE METHOD FOR DETERMINING THE PARITY OF THE CASCADE HYPERON. Saul Barshay, Physics Department, Brandeis University, Waltham, Massachusetts (Received April 25, 1960).

A study of the reaction $\bar{\Xi}^+ + p \rightarrow K^+ + K^+$ is suggested as a possible means of determining the parity of the cascade hyperon relative to the nucleon.

INTERACTION BETWEEN A PAIR OF K^+ MESONS. Saul Barshay, Physics Department, Brandeis University, Waltham, Massachusetts (Received April 25, 1960).

An analysis is performed for the reaction $\bar{\Xi}^+ + p \rightarrow K^+ + K^+ + \gamma$, with subsequent materialization of the photon into an electron-positron pair. In the circumstance that (a) the annihilation of low-energy $\bar{\Xi}^+$ proceeds largely via the reaction $\bar{\Xi}^+ + p \rightarrow K^+ + K^+$ and that (b) the elastic scattering of the $\bar{\Xi}^+$ on hydrogen is largely a diffraction effect, a study of the radiative reaction provides, in principle, a means of determining

a function related to the phase shifts for the scattering of a pair of K^+ mesons. Under assumption (b) alone, an absence of all meson-meson interactions among pions and K mesons could be inferred from the vanishing of a certain angular correlation term in the radiative process.

NUCLEON-NUCLEON SPIN-ORBIT INTERACTION AND THE REPULSIVE CORE. G. Breit, Yale University, New Haven, Connecticut (Received May 27, 1960).

Recent proposals to explain the phenomenological repulsive core and spin-orbit interaction in nucleon-nucleon scattering in terms of a vactor meson field are discussed. Estimates of the mass of the vector meson made on the basis of the Bryan potential may need some revision on account of insufficiently studied possibilities of modifying that potential. Estimates of interaction constants on the basis of the Signell-Zinn-Marshak potential and the replacement of a two-body relativistic problem by a one-body problem do not appear applicable. Estimates based on a covariant matrix element but neglecting wave function distortion in the analysis of 300-Mev data are shown to be quite uncertain. Accordingly the evidence for a vector meson mass of $3m_\pi$ or $4m_\pi$ also appears to have little weight.

Masses $3m_\pi$ and $4m_\pi$ are shown to lead to central-field potential energy tails which extend into the one-pion-exchange potential region and appear therefore to be improbably large. They also lead to repulsive cores which do not fit in with the usual phenomenological hard cores as naturally as the larger heavy-photon masses. Brief mention is made of possible means of detecting the vector meson and of the effects of its finite mean life.

$K\pi$ INTERACTION IN THE DOUBLE DISPERSION REPRESENTATION. Benjamin W. Lee, Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania (Received April 27, 1960; revised manuscript received June 22, 1960).

The low-energy $K\pi$ interaction is studied on the basis of the double dispersion representation. Exact dispersion relations for partial wave amplitudes are derived for $K\pi$ scattering and for the process $\pi+\pi\rightarrow K+\bar{K}$. These relations are reduced to manageable form and effective-range formulas are derived under the assumption that the $K\pi$ interaction proceeds principally through a zero-range "potential." For the process $\pi+\pi\rightarrow K+\bar{K}$, the initial state interaction is taken into account; for the p wave of $K\pi$ scattering, the two-pion exchange mechanism is considered. Other types of solutions are briefly discussed.

PION-PION INTERACTION IN ELECTROMAGNETIC PROCESSES. Laurie Brown,* Istituto di Fisica dell'Università, Roma, Italia and Francesco Calogero, Istituto di Fisica dell'Università, Roma, Italia and Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Italia (Received May 3, 1960; revised manuscript received June 1, 1960).

The correction to the photon propagator due to the emission of a virtual pion pair is evaluated taking into account the effect of the strong pion-pion interaction in the $J=1$, $T=1$ state of the pions recently discussed by several authors. Results are given for different sets of parameters describing the pion form factor, and the possibility of investigating the structure of the form factor by means of electron-electron and electron-positron collision experiments is studied.

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DIVERGENCE-FREE ITERATIVE EXPANSION OF THE S MATRIX IN A FIELD THEORY. Kurt Haller, New York University, New York, New York (Received March 30, 1960).

A new method is proposed for evaluating the S matrix as a series expansion in powers of the coupling constant. The method is applicable to field theories which, in the usual formulation, have ultraviolet divergences in self-energy and vertex parts and require self-energy, coupling constant, and wave-function renormalization. The procedure cannot be applied in its present form to theories which allow boson self-energy terms. In this new procedure the usual form of the Hamiltonian for the coupled system is retained. The theory results in an iterated solution in powers of the physical coupling constant and yields a series, each term of which is finite without subtractions or renormalization. It

agrees up to all orders examined with the finite S-matrix elements obtained by renormalizing the old formulation of the scattering problem.

It is also shown that the nth order contribution to the iterative expansion of the S matrix, where n is any order, approaches 0 more rapidly, at high energy, than $E^{-(2-\delta)}$, where δ is any positive real number, no matter how small.

PROTON-PROTON EFFECTIVE RANGE THEORY WITH VACUUM POLARIZATION. Leon Heller, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received May 5, 1960; revised manuscript received June 20, 1960).

The effect of vacuum polarization upon p-p scattering is considered by first solving the problem in the Coulomb plus vacuum polarization potentials (called "electric" potential) without the nuclear potential. The nuclear phase shifts are then defined with respect to the electric wave functions, and the scattering cross section is written in terms of these phase shifts. The connection with other nuclear phase shifts (in the presence of vacuum polarization) which appear in the literature is established.

The effective-range expansion for the nuclear s-wave phase shift is derived. An analysis of three low-energy p-p scattering experiments indicates that the omission of vacuum polarization from the analysis results in a value for the shape-dependent parameter which is 0.02 smaller than the value obtained when vacuum polarization is included in the analysis. This latter value is 0.04_7 with, however, an uncertainty $|\delta P| > 0.02_5$. A discussion of the accuracy required to usefully delimit the parameter P is included.

GRAVITATIONAL-ELECTROMAGNETIC COUPLING AND THE CLASSICAL SELF-ENERGY PROBLEM. R. Arnowitt, Department of Physics, Syracuse University, Syracuse, New York, S. Deser, Department of Physics, Brandeis University, Waltham, Massachusetts, and C. W. Misner, Palmer Physical Laboratory, Princeton University, Princeton, New Jersey (Received April 21, 1960).

The gravitational effect on the classical Coulomb self-energy of a point charge is calculated rigorously. It is shown that the total mass then becomes finite (although still quite large), and that it depends only on the charge and not on the bare mechanical mass. Thus, a particle acquires mass only when it has nongravitational interactions with fields of nonzero range. In order to treat this problem, it is necessary to extend the canonical formalism, previously obtained for the free gravitational field, to include coupling with the Maxwell field and the point-charge system. It is shown that the canonical variables of the gravitational field are unaltered while those of the matter system are natural generalizations of their flat-space forms. The determination of the total energy of a state can still be made from knowledge of the spatial metric at a given time. The self-mass of a particle is then the total energy of a pure one-particle state, i.e., a state containing no excitations of the canonical variables of the Maxwell or Einstein fields. Solutions corresponding to pure particle states of two like charges are also obtained, and their energy is shown to be consistent with the one-particle results.

INTERIOR SCHWARZSCHILD SOLUTIONS AND INTERPRETATION OF SOURCE TERMS. R. Arnowitt,* S. Deser, and C. W. Misner,† Department of Physics, Brandeis University, Waltham, Massachusetts (Received April 27, 1960).

The solutions of the Einstein field equations, previously used in deriving the self-energy of a point charge, are shown to be nonsingular in a canonical frame, except at the position of the particle. A distribution of "dust" of finite extension is examined as the model whose limit is the point particle. The standard "proper rest-mass density" is related to the bare rest-mass density. The lack of singularity of the initial metric $g_{\mu\nu}$ is in contrast to the Schwarzschild type singularity of standard coordinate systems. Our solutions for the extended source are nonstatic in general, corresponding to the fact that a charged dust is not generally in equilibrium. However, the solutions become static in the point limit for all values of the bare source parameters. Similarly, the self-stresses vanish for the point particle. Thus, a classical point electron is stable, the gravitational interaction cancelling the electrostatic self-force, without the need for any extraneous "cohesive" forces.

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MEASUREMENT OF QUANTUM MECHANICAL OPERATORS. Huzihiro Araki* and Mutsuo M. Yanase,† Palmer Physical Laboratory, Princeton University, Princeton, New Jersey (Received May 17, 1960).

The limitation on the measurement of an operator imposed by the presence of a conservation law is studied. It is shown that an operator which does not commute with a conserved (additive) quantity can not be measured exactly (in the sense of von Neumann). It is also shown for a simple case that an approximate measurement of such an operator is possible to any desired accuracy.

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